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ABSTRACT

This document is the student manual for the learning module, "Teaching the Concept of Area." The purpose of the module is to enable the student to present the underlying concepts of area in a meaningful and useful way. The terminal objective of the module is to be able to demonstrate an understanding of the concept of area, to construct a sequence of learning tasks appropriate for ten- to twelve-year-old students, to present these tasks to such a group, and to assess their performance. The module consists of three sections, the first containing a pre-assessment and seven elements/activities, with post-assessments for each. The second section contains activities useful in constructing a learning sequence for ten- to twelve-year-old students, and the third section presents directions for implementing and assessing the learning sequence. Instructor's manual and response manual available separately. (MB)

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TOWARD COMPETENCE

INSTRUCTIONAL MATERIALS FOR TEACHER EDUCATION

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COMPETENCY BASED TEACHER EDUCATION PROJECT
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SP010 502

A MODULE APPROACH

INTEGRATING CONTENT, METHOD, AND FIELD EXPERIENCE

TEACHING THE CONCEPT OF AREA

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WHAT IS COMPETENCY-BASED TEACHER EDUCATION?

The set of materials you are about to begin using represents a new direction in teacher education. Called competency-based teacher education, this approach to training teachers emphasizes the teacher's performance--what he or she is actually able to do as the result of acquiring certain knowledge or skills.

Performance in a specific area is referred to as a competency. Thus, what we expect the teacher to be like after completing his education can be described in terms of the competencies he should have. The emphasis is on doing rather than on knowing, though performance is frequently the result of knowledge.

This shift in emphasis from knowing to doing accounts, to a great extent, for the differences you will notice in the format and content of these materials. To begin with, the set of materials itself is called a module because it is thought of as one part of an entire system of instruction. The focus has been narrowed to one competency or to a small group of closely related competencies. The ultimate aim of the module is expressed as a terminal objective, a statement describing what you, the teacher, should be able to do as a result of successfully completing this module.

Your final performance, however, can usually be broken down into a series of smaller, more specific objectives. As you achieve each of these, you are taking a step toward fulfilling the ultimate goal of the module. Each intermediate objective is the focus of a group of activities designed to enable you to reach that objective.

Together, the activities that make up each element, or part, of the module enable you to achieve the terminal objective.

There are several kinds of objectives, depending on what kind of performance is being demanded of you. For example, in a cognitive-based objective, the emphasis is on what you know. But since these are behavioral objectives, what you know can only be determined overtly. An objective can only be stated in terms of your behavior--what you can do. You might, for instance, be asked to demonstrate your knowledge of a subject by performing certain tasks, such as correctly completing arithmetic problems or matching words and definitions. In addition to cognitive-based objectives, there are performance-based objectives, where the criterion is your actual skill in carrying out a task; consequence-based objectives, for which your success in teaching something to someone else is measured; and exploratory objectives, which are open-ended, inviting you to investigate certain questions in an unstructured way.

Along with the assumption that the competencies, or behaviors, that make for successful teaching can be identified goes the assumption that these competencies can be assessed in some way. In fact, the statement of objectives and the development of assessment procedures form the main thrust of competency-based teacher education. The module, and the activities it contains or prescribes, is just a way of implementing the objectives.

But the module does have certain advantages as an instructional tool. For one thing, it enables you to work on your own and at your own pace. The activities are usually varied so that you can

select those which are best suited to your learning style. And the module enables you to cover certain subject areas with maximum efficiency; since if you pass the pre-assessment for a given objective, you are exempted from the module implementing that objective. What matters is not the amount of classroom time you put in on a subject but your ability to demonstrate certain competencies, or behaviors.

OVERVIEW

Mathematics is usually thought of as a study of number relationships. However, the learning of computational techniques is but a small part of the mathematical experiences that children need to have in order to function effectively in the real world. The concept of space and ways of measuring it have been frequently limited to meaningless formulas. The study of area, one aspect of space, is rich in practical applications and is particularly significant as a building block for future mathematical study.

Although you, the reader, may think of finding the area of a region as a series of formulas, there are many underlying concepts which are necessary to make those formulas meaningful and useful. This module is designed to help you develop an intuitive understanding of the concept of area, as applied to a variety of surfaces and to become familiar with alternative strategies needed to teach the concept to children.

Upon completing this module, you should be able to:

1. Demonstrate an understanding of the concept of area in each of the following ways:

   a. Given a set of representative materials, classify the materials according to their appropriateness for measuring 1, 2, or 3 dimensional space and justify the material selection for measurement of 2 dimensional space.

   b. Given a set of shapes, select suitable shapes as units for completely covering a given surface area.

   c. Given selected geometric shapes, find their areas in different standard and non-standard units of measure.

   d. Given a region, determine change in area as the dimensions are varied.

   e. Given a series of experiments, identify those that probe conservation of area.

   f. Given a set of materials, order them and justify that they lead to a discoverable pattern; and demonstrate with these materials how to derive the formulas for the areas of selected geometric figures.

   g. Given a set of geometric figures, restructure these figures to demonstrate the equivalence of areas.

h. Given problem situations, identify where the concept of area is involved and can produce solutions to the problems.

2. Given a set of specific objectives for developing the concept of area, construct sequences of learning tasks that can be used for ten- to twelve-year-old students.

3. Given one of these sequences of learning tasks that you have developed, present it to a group of ten-to twelve-year-old students and then analyze and assess its appropriateness.

Prerequisites

Since the module involves teaching mathematical concepts to a group of fifth- and sixth-grade pupils, you will need to have access to students in order to complete the tasks. A pre-service student who wants to specialize in mathematics, a graduate student in mathematics education, and an in-service elementary teacher could all benefit from this module.

As a help in scheduling your time, think of this module as a six-week mini course.

How to Take this Module

The module is divided into three section plus a separate pre-assessment instrument. Your responses to the pre-assessment test will determine which activities of Section I you need to complete in order to help you develop complete understanding of this concept. If you complete the pre-assessment successfully, do all the activities in Sections II and III.

If, after you have read the pre-assessment test, you feel that you cannot answer most of the questions, you may, if you choose, skip the pre-assessment test and proceed immediately to page 13 and do all the elements in Section I in order to develop the competencies you lack.

Each element includes several tasks so that you not only become familiar with many facets of the concept but also with a variety of learning strategies.

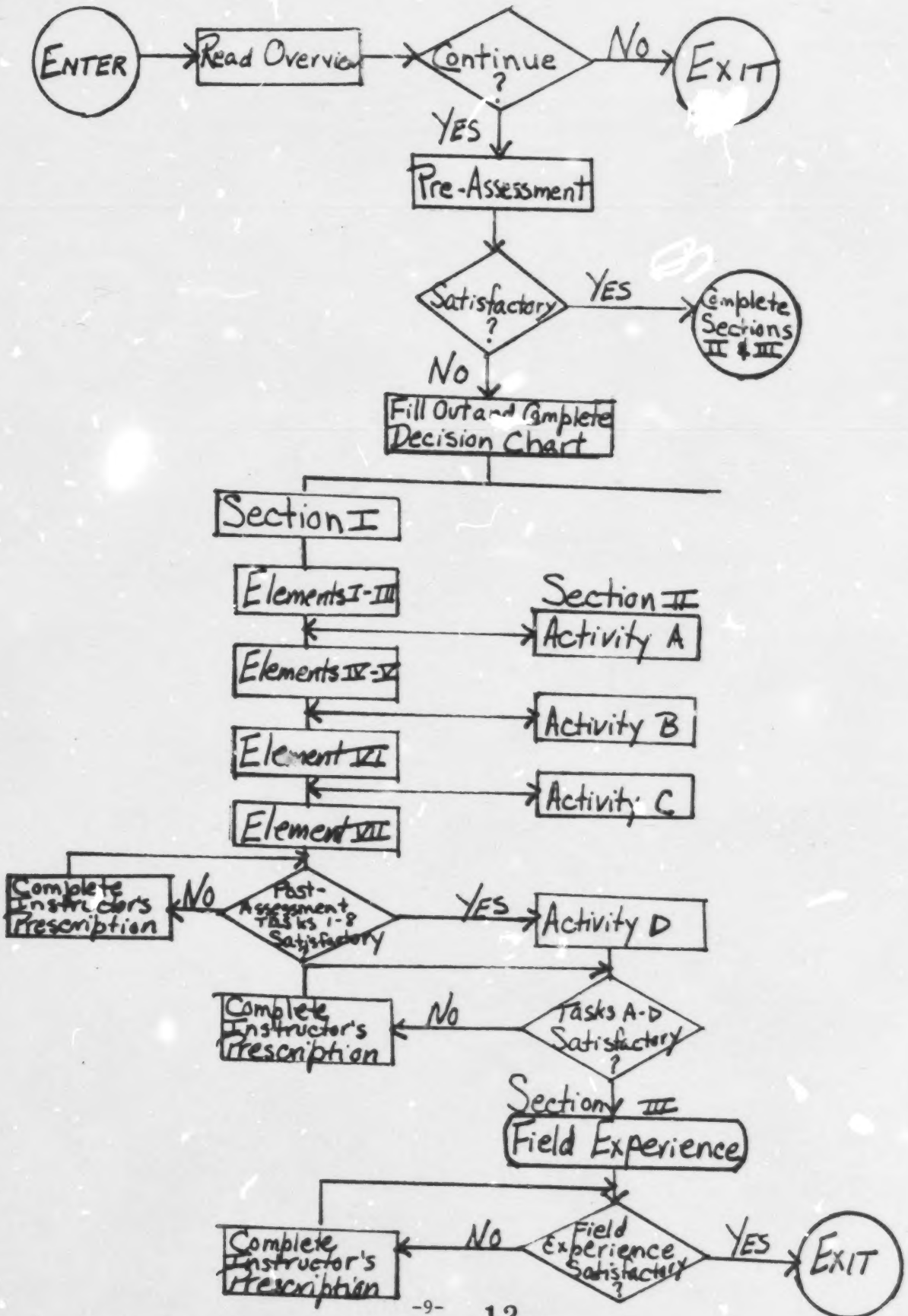
Occasionally a bit more challenging problem is included among the tasks and it will be starred. You can check your responses to all the tasks by consulting the sections entitled "Discussion of Responses" provided with each activity. There are Post-Assessment Tasks for each activity which you must complete satisfactorily. "Satisfactory Completion" implies that at least 90% of your responses to the problems are correct.

Section II of the module contains a set of activities in which you develop sequences of learning tasks to be used in teaching the concept of area to children. Each activity of Section II is to be done after you have completed specific activities in Section I. To complete the module you must do all the activities in Section II.

In Section III of this module, you will teach a mini-lesson on the concept of area to a group of children using one of the sequences of learning tasks that you developed in Section II. Following your presentation of the lesson, you will analyze and

assess its appropriateness.

On the next page you will find a flow chart which diagrams the paths you may follow in order to complete this module. After you have examined the flow chart, either take the pre-assessment or begin Element I on page 15. If you decide to take the pre-assessment, your instructor will score it, return it to you, and help you fill out the Decision Chart on page 10.


ENTER
Read Overview
Continue ?
No
EXIT
YES
Pre-Assessment
Satisfactory ?
YES
Complete Sections II & III
No
Fill Out and Complete Decision Chart
Section I
Elements I-III
Elements IV-V
Element VI
Element VII
Section II
Activity A
Activity B
Activity C
Complete Instructors Prescription
No
Post-Assessment Tasks 1-8 Satisfactory
YES
Activity D
Complete Instructor's Prescription
No
Tasks A-D Satisfactory ?
Section III
Field Experience
Complete Instructor's Prescription
No
Field Experience Satisfactory ?
YES
EXIT

DECISION CHART

I Circle the problem numbers which your instructor scored as correct	II Directions for you	Place a check in this column when you satisfactorily complete activities in Section I	Place a check in this column when your instructor approves your tasks in Section II
1. ----------→	Go to Element I, page 15.		
2. ----------→	Go to Element II, page 28.		
3. ----------→	Go to Element III, page 41.		
(A) -------→	Go to Activity A, page 189.		
4. ----------→	Go to Element IV, page 63.		
5. ----------→	Go to Element V, page 92.		
6. ----------→	Go to Element IV, page 63, and to Element VI, page 119.		
7. ----------→	Go to Element V, page 92.		
(B) -------→	Go to Activity B, page 192.		
8. ----------→	Go to Element VI, page 119.		
9. ----------→	Go to Element VI, page 119.		
(C) -------→	Go to Activity C, page 195.		
10. ----------→	Go to Element VII, page 161.		
(D) -------→	Go to Activity D, page 201.		

Directions:

1. For your scored responses on the Diagnostic Pre-Assessment Test, circle in column I on your Decision Chart on page 10 the problem numbers which your instructor scored as incorrect.

2. For each of the problem numbers circled in column I, there are directions given in column II for a specific activity to develop the competencies that you lack. Each of the activities has several tasks which you must complete.

3. You must do all the activities associated with each of the circled items in column I of your Decision Chart. You will note that all the activities of Section II have been circled.

4. Proceed through the activities associated with the circled items in chronological order as they appear on your Decision Chart.

5. As you satisfactorily* complete each of the activities associated with the circled items, place a check in the appropriate column.

6. When you have satisfactorily completed all the required activities, proceed to Section III.

7. For specific mathematical terms with which you may be unfamiliar, refer to the GLOSSARY OF TERMS on pages 180-181 at the end of Section I. You may refer to these definitions as you need them.

You are now ready to begin the activities noted on your Decision Chart. Go to the first circled item in column I and follow the directions in column II.

Have you satisfactorily completed all the required activities to each circled item?

Yes_____

No_____

If yes, proceed to Section III, page 207.

If no, complete the remaining required activities.

*The phrase "satisfactory completion" implies that at least 90% of your reponses to the problems (excluding starred problems) are correct.

Section I

ELEMENT I

Objective: Given a set of representative materials, you will be able to classify them correctly according to their appropriateness for measuring one-, two-, or three-dimensional space and justify the material selection for measurement of two-dimensional space.

This element is designed to help you understand the classification of materials needed to measure two-dimensional space. In order to achieve the goal of this element you will need to know something about the terms used in describing space. This element begins, therefore, with a short background section that will introduce these terms to you and give you practice in applying them. After you have become familiar with the terms and ideas you will be working with, you are given a choice of completing tasks that are specially written for this module or completing the reading of three selections from a book list. The tasks are found in Activity 1, which begins on page 19, and the book list, Activity 2, is on page 25. When you have finished your chosen Activity, turn to page 26 for the post-assessment for this element. Satisfactory completion of the post-assessment means that you have met the objective of this element and may go on to the next circled element in your Decision Sheet.

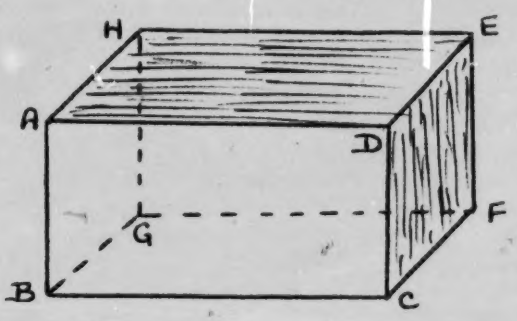
Before you choose your Activity, read through these next few pages.

Background

1. What is a two-dimensional space?

A two-dimensional space is a plane region*. The idea of a plane region in geometry is suggested by a flat surface. Any flat surface such as a wall of the room, the floor, the top of a desk, is a good representation of a plane. The geometric concept of a plane allows for only two dimensions, while the physical representation of a plane, i.e., the top of a desk, cannot avoid getting into three dimensions. However, there is no thickness or depth to a plane. Hence, the concept of "plane"--the two-dimensional surface--is one we can idealize, that is, have an idea of, but never actually encounter.

The figure below is a diagram of a room.



AHED is a plane region which represents the ceiling.

CDEF is a plane region which represents a wall.

Now you try identifying some planes.

1. Name the four plane regions which represent the walls of the room -

_____, _____

_____, _____

* Mathematical definitions for words or phrases underlined in text are given in the Glossary, pages 180-181.

2. Name the plane region which represents the floor -

_____

Compare your answers with the following:

1. The four wall plane regions:

   AHGB, HGFE, DEFC, ADCB

2. The floor is represented by the plane region

   BGFC

2. How do you define the measure of a geometric object?

The measure of a geometric object (line segment, perimeter, plane region) in terms of a unit is the number (not necessarily a whole number) of times the unit will fit into the object.

For example, if you wished to measure how long your kitchen table is, you would take a ruler and count the number of matching units. Your answer might be 30 where the measuring unit is an inch or your answer might be 2 1/2 where the measuring unit is a foot. On the other hand, you might wish to find the measure of the top surface of the kitchen table. You find it can be covered with four sheets of 12" x 15" paper. Your answer then, for the measure of top surface might be four where the measuring unit is a piece of 12" x 15" paper.

Here are two examples for you to try:

1. With your ruler, using a centimeter as your unit, measure the length and width of this page.

   Length_____

   Width_____

2. How many 3" x 5" index cards would you need to cover this page?

   Answer_____

Compare your results with these:

1. Length = 28 centimeters

   Width = $21\frac{3}{4}$ centimeters

2. Approximately $6\frac{1}{4}$ index cards

NOW TURN TO THE NEXT PAGE IF YOU HAVE CHOSEN ACTIVITY 1 AND TO PAGE 25 IF YOU HAVE CHOSEN ACTIVITY 2.

Activity 1

If you have chosen this activity, you will be asked to complete two tasks and check them yourself. If you have any problems or feel that you need more help, ask your instructor for guidance.

Task 1.1. The figure on page 20 is a picture of a room. Select as many objects in the room that you can find which are classified as plane regions and list them below.

Give reasons for your answers.

TURN TO PAGE 21 AND CHECK YOUR RESPONSES.

Discussion of Responses:

Accepted responses are many.

Your responses are to include plane region shapes, such as rug, top of TV, front of TV, top of box on the table, the sides of that box, the top of the table, the side of the chair, the pictures on the wall, the wall.

Have you completed Task 1.1 satisfactorily? *

Yes - Proceed to Task 1.2.

No - Is this your first try?

Yes - Read one of the suggested texts on page 25 and redo exercise.

No - See instructor.

---

* Satisfactorily, in this case, implies 75 - 80% correct responses.

Task 1.2. Each closed figure in a plane is comprised of a closed curve and its interior. Both the closed curve and the interior may be measured. To measure the closed curve you measure the length of the curve. This is a one-dimensional measurement. To measure the interior of the closed curve you find the number of units needed to cover the region. This is a two-dimensional measurement. The measure of a region is called its area.

Tell whether you are interested in the length of a simple closed curve or covering the interior, or both[1] and tell whether it is one dimensional or two dimensional:

1. To trim the edge of a handkerchief with lace.

   _____

2. To buy a rug to cover the living room floor.

   _____

3. To buy a desk blotter for your desk.

   _____

4. To put a book cover on your text.

   _____

5. To string enough beads for a necklace.

   _____

6. Can you give 3 other examples of situations in which you would need to know the size of the surface enclosed by a simple closed curve rather than just the length of the curve?

   _____

   _____

   _____

TURN TO THE NEXT PAGE AND CHECK YOUR RESPONSES.

---

[1] SMSG, Grade 5, p. 766. See Bibliography for complete listing.

Discussion of Responses:

1. Length of curve - one dimensional

2. Length of curve - one dimensional
   Covering the interior - two dimensional

3. Length of curve - one dimensional
   Covering the interior - two dimensional

4. Length of curve - one dimensional
   Covering the interior - two dimensional

5. Length of curve - one dimensional

6. Answers will vary, but your answers should be similar to these suggestions:

   a. To buy material to make a slipcover for a chair

   b. To fit a table top with a glass cover

   c. To buy linoleum for a kitchen floor

Have you completed Task 1.2 satisfactorily?

Yes - Proceed to Post Assessment for Element 1, page 26.

No - Is this your first try?

Yes - Read one of the suggested texts on page 25 and redo exercise.

No - See instructor.

Activity 2

From the following list of readings, select three and study the indicated pages or chapter. When you have finished the reading, turn to the post-assessment for Element I on page 26 and follow the directions given there.

1. Copland, Mathematics and the Elementary Teacher, Chapter 10.

2. Fitzgerald,et al, Laboratory Manual Elementary Mathematics, Second Edition, pp. 103-109.

3. Elliott, et al, Geometry in the Classroom, pp. 63-74.

4. SMP, Book B., pp. 36-44.

5. Young and Bush, Geometry for Elementary Teachers, pp. 11-117.

6. Lovell, The Growth of Understanding in Mathematics, pp 107-114.

7. Schminke, Maertens, Arnold, Teaching the Child Mathematics, pp. 337-350.

Post-Assessment

Suppose you were given a box which contained the following items: string, a reel of wire, chicken wire, a ruler, coins, and dry kidney beans, water, tiles, cubes, cups, marbles, index cards, stamps, sheets of paper, board erasers, bricks, books, foot prints, leaves, a roll of brown gummed tape and a roll of cotton 2" wide gauze (bandage).

Classify at least 20 of these items according to their appropriateness for measuring one-, two-, or three-dimensional space. Tell whether you are interested in the length of a simple closed curve or covering the interior or both in the type of measurement column. If you have doubts about how to classify the items, reread the instructions of Task 1.2.

TABLE I

Name of Item	Type of Measurement
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	
11.	
12.	
13.	
14.	
15.	
16.	
17.	
18.	
19.	
20.	

Justify your selection of the items for measurement of two-dimensional space.

_____

_____

_____

_____

Directions

1. Check your responses to the post-assessment on pages 1-2 of the RESPONSE MANUAL.

2. Score of Your Responses to Table I

   Number Correct_____

   Number Incorrect_____

   Does your explanation of your selection of the measurement of two dimensional space agree with that of the Response Manual?

   Check one Yes_____ No_____

3. If you have four or more responses from Table I incorrect and you have checked No to the explanation question, read one of the suggested readings given on page 25 and redo the task. When you have redone the task satisfactorily, go to direction 5 and complete the directions.

4. If you have sixteen or more responses correct, and you have checked Yes to the explanation question, go back to page 10.

5. On your Decision Chart, place a check in column 3 to indicate that you have completed Element I.

6. Go to the next circled item. Follow the directions given for that item.

ELEMENT II

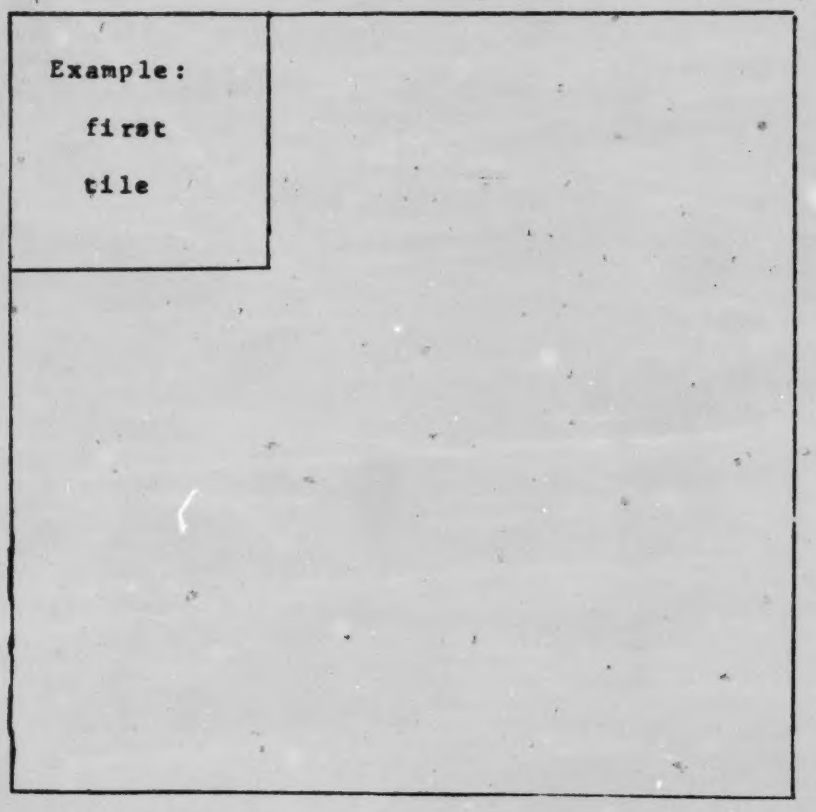
Objective: Given a set of shapes, you will select those suitable as units for completely covering a given surface area.

This element is designed to enable you to select shapes that can be used as units for completely covering a given surface. Like all the elements, it begins with a short introduction to the terms and concepts you will be using. After you have become familiar with these concepts, you can choose to complete either Activity 1--a series of tasks specially written for this module--or Activity 2--outside readings. When you have finished your chosen activity, turn to page 39 for the post-assessment for this element. Satisfactory completion of the post-assessment means that you have met the objective of this element and may go on to the next circled element in your Decision Sheet.

Many of the ancients decorated their buildings and homes with mosaic floors and pavements made of very small tiles called tessellae. The tiles were placed side by side to cover the floors and pavements completely. The word tessellation is used to describe ways of filling space. This filled space is a space entirely covered with the same size units placed side by side with no overlapping of the unit.

For example: Your kitchen or bathroom floor may be covered with tiles all of the same size or shape.

Try the following: Trace the sample tile at the right and cut it out. Then see how many times you can place it side by side in order to cover completely the rectangular region on the next page.



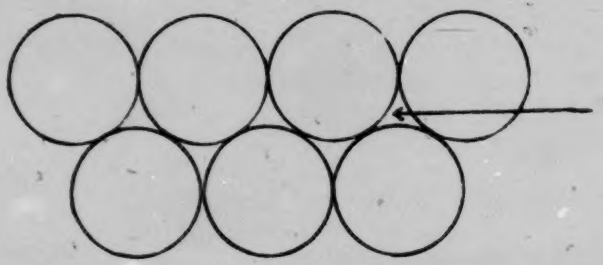
It should take nine of the sample tiles placed side by side to cover the region.

This resulting pattern of 9 tiles is called a tessellation and the sample tile is said to have the property of being able to tessellate since a region can be completely covered by using it repeatedly.

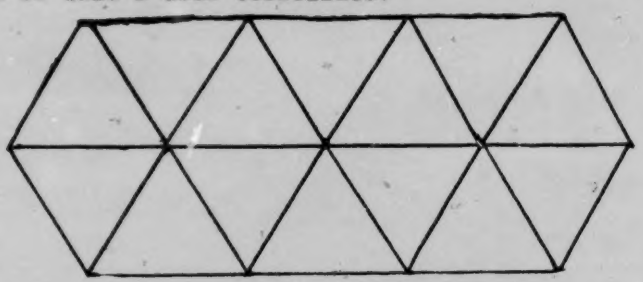
Consider the following sample tiles A and B:

Explore the possibility that each will tessellate. Select a small region about 4" x 4" to work with. Trace or cut out each tile and try to cover your region completely first with one, then with the other.

It is clear that Tile A does not tessellate since it is not possible to cover the entire region with the circles.



However, it is possible to cover a plane region completely with Tile B and so Tile B does tessellate.



(Note: It is assumed the plane region is not bounded.)

NOW CONTINUE ON THIS PAGE IF YOU HAVE CHOSEN ACTIVITY 1 AND TURN TO PAGE 31 IF YOU HAVE CHOSEN ACTIVITY 2.

Activity 1

If you have chosen this activity, you will be asked to complete two tasks and check them yourself. If you have any problems or feel that you need more help, ask your instructor for guidance.

Before you start the tasks, see that you have the following materials at hand:

Tracing paper
Scissors
Ruler
Unlined paper
Book--Tessellations, Josephine Mold

Activity 2

From the following list of readings, select three and study the indicated pages. When you have finished the reading, turn to the post-assessment for Element II on page 39 and follow the directions given there.

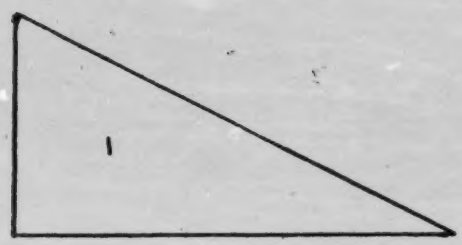
1. Elliott, et al, Geometry in the Classroom, pp. 75-77, pp. 147-150.

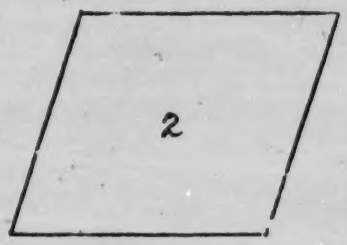
2. Mold, Tessellations.

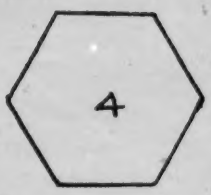
3. Pearcy and Lewis, Experiments in Mathematics, pp. 14-15.

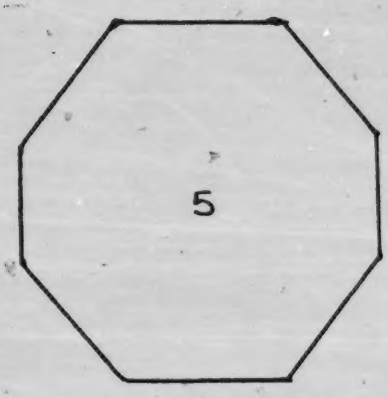
4. SMP, Book B, pp. 13-18.

Task 2.1. For each of the unit shapes below, explore the possibility of the unit shape tessellating. Trace each shape and cut it out. Then for each shape try to form a Mosaic Pattern which covers a region completely. Record in Table 2.1 on the next page whether or not the unit shape tessellates.









For each of the unit shapes given below, tell whether the unit shape did or did not tessellate by writing yes or no in the blank.

TABLE 2.1

Unit Shape	Yes or No
1. Right Triangle	
2. Parallelogram	
3. Pentagon	
4. Hexagon	
5. Octagon	

Indicate reasons why some of the units did not tessellate.

TURN TO THE NEXT PAGE AND CHECK YOUR RESPONSES.

Discussion of Responses:

TABLE 2.1

Unit Shape	Yes or No
Right Triangle	Yes
Parallelogram	Yes
Pentagon	No
Hexagon	Yes
Octagon	No

Reasons for the units which did not tessellate may include:

1. The region could not be entirely filled unless overlapping was allowed. Examples, the circle and pentagon.

2. The units could no be placed side by side to entirely cover the region.

Have you completed Task 2.1 satisfactorily?

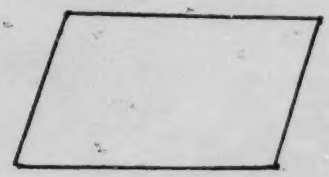
Yes - Proceed to Task 2.2.

No - Is this your first try?

Yes - Read one of the suggested texts on page 31 and redo exercise.

No - See instructor.

Task 2.2 There are many different designs which can be used to get a curved effect in a tiling pattern. Whenever you turn your tessellations into something curved you must remember whatever you take from one side must be attached to an opposite side. For example, in Task 2.1, you saw that a parallelogram tessellates. Now modify the parallelogram shape as indicated below:



parallelogram

modified parallelogram

a. Trace the modified parallelogram and cut it out. In the space below see if it tesselates, i.e., try to create a Mosaic Pattern to cover the space completely using only the modified parallelogram.

b. Turn to page 28 of the book Tessellations, by Josephine Mold. In the space below draw two equilateral triangle tiles which give a tessellation of a single shape with curved edges when fitted together.

c. Trace and cut out the two motifs on page 30 of Tessellations. In the space provided below, complete the exercise.

c. (continued) Additional space to complete the exercise.

d. Invent other curved shapes which will tessellate. Record them below:

TURN TO THE NEXT PAGE AND CHECK YOUR RESPONSES.

Discussion of Responses:

This task is designed to let you discover the tessellating of curved units. Your responses may be checked by reading Tessellations by Josephine Mold, pages 28-31.

Have you completed Task 2.2 satisfactorily?

Yes - Proceed to Post-Assessment for Element II, page 39.

No - Is this your first try?

Yes - Read one of the suggested texts on page 31 and redo exercise.

No - See instructor.

Post-Assessment

Complete the table below based on Exercise 8 in SMP, Book B, page 4.

TABLE II

Figure	Yes or No
a.	
b.	
c.	
d.	
e.	
f.	
g.	
h.	
i.	
j.	
k.	
l.	
m.	
n.	
o.	
p.	

Directions

1. Check your responses to the post-assessment on page 3 of the RESPONSE MANUAL.

2. Score of Your Responses to Table II

   Number Correct_____

   Number Incorrect_____

3. If you have four or more responses incorrect, read one of the suggested readings given on page 31 and redo the task. When you have redone the task satisfactorily, go on to Direction 4 and continue.

4. If you have thirteen or more responses correct, go back to page 10.

5. On your Decision Chart, place a check in column 3 to indicate that you have completed Element 2.

6. Go to the next circled item. Follow the directions given for that item.

ELEMENT III

Objective: Given selected geometric shapes you will be able to find their areas using both standard and non standard units of measure.

This element is designed to help you find the area of a variety of different shapes using a variety of different units of measure. Like all the elements, it begins with a short introduction to the terms and concepts you will be using. After you have become familiar with these concepts, you can choose to complete either Activity 1--a series of tasks specially written for this module--or Activity 2--outside readings. When you have finished your chosen activity, turn to page 61 for the post-assessment for this element. Satisfactory completion of the post-assessment means that you have met the objective of this element and may go on to the next circled element in your Decision Sheet.

It is very reasonable for a man to survey his property and ask, "How much land do I own?" "Is my land larger than my neighbor's land?" The man needs some basic unit with which to measure the regions. The shape of the unit is arbitrary. It may be triangular, hexagonal, or rectangular. The shape is selected on the basis of convenience. The area of the region is stated in terms of the number and kind of selected shapes.

For example, consider the following:

How many 3 x 5 index cards are needed to cover Region A (on next page) completely with no overlapping?

1. The measure of the page is _____

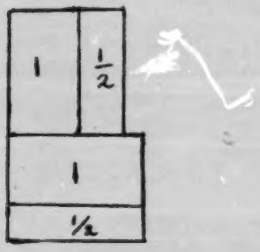
2. The unit of measure is _____

42

Your responses should be:

1. 3 index cards

2. index card



How many times would you have to put a 10-cent stamp on Region A with no overlapping to cover the region completely?

a. The unit of measure is_____

b. The measure of Region A is _____

For a your unit of measure is a 10-cent stamp. The answers for b will vary depending upon the size of 10-cent stamp you used

The area of Region A when using an index card as a unit was 3. The area of Region A when using a 10-cent stamp as a unit was much larger (see your answer to b above).

Why do the answers differ?

Clearly, even though it is the same region that is covered, the answers differ because the units are different in size.

TURN TO THE NEXT PAGE IF YOU HAVE CHOSEN ACTIVITY 1, AND TO PAGE 60 IF YOU HAVE CHOSEN ACTIVITY 2.

Activity 1

If you have chosen this activity, you will be asked to do two tasks and check them yourself. If you have any questions or think you need further guidance, ask your instructor for help.

Before you start the tasks, see that you have the following materials on hand:

Index cards - 3 x 5
Postage stamps - 10¢ size
Ruler - inches
Ruler - centimeters
Unlined paper
Graph paper
SMP, Book B
Cuisenaire rods
Tracing paper

Task 3.1. This activity requires a set of Cuisenaire rods.

a. Using one face of a white rod as a stamp, how many times would you have to stamp an orange rod to cover it completely by stamping?

Answer_____

b. Using the same stamping process as the above to cover the indicated rod, how many times would you have to stamp:

1. a blue rod _____
2. a brown rod _____
3. a black rod _____
4. a dark green rod _____
5. a yellow rod _____
6. a purple rod _____
7. a light green rod _____
8. a red rod _____

c. If you form a "train" of a blue rod, a yellow rod, and a light green rod placed end to end, how many times would you have to stamp the "train" with the white rod to cover it completely?

Answer_____

d. If one of the non-square faces of a red rod was used as a stamp, how many times would you have to stamp an orange rod to cover it without covering the ends of the orange rod?

Answer_____

TURN TO THE NEXT PAGE AND CHECK YOUR RESPONSES.

Discussion of Responses:

a. 42 times

b. 1. 38 times

2. 34 times

3. 30 times

4. 26 times

5. 22 times

6. 18 times

7. 14 times

8. 10 times

c. 70 times

d. 20 times

Have you completed Task 3.1 satisfactorily?

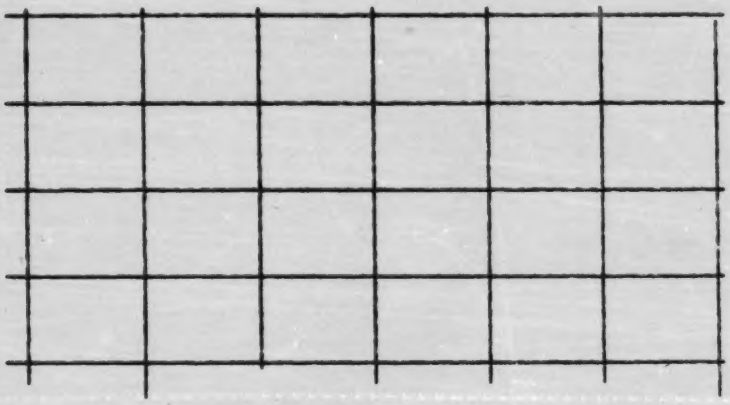
Yes - Proceed to Task 3.2

No - Is this your first try?

Yes - Read one of the suggested texts on page 60 and redo exercise.

No - See instructor.

Task 3.2.[1] A grid is a convenient tool to measure with. It is formed by the boundaries of the units used in a tessellation. The unit of the grid below is a rectangle.

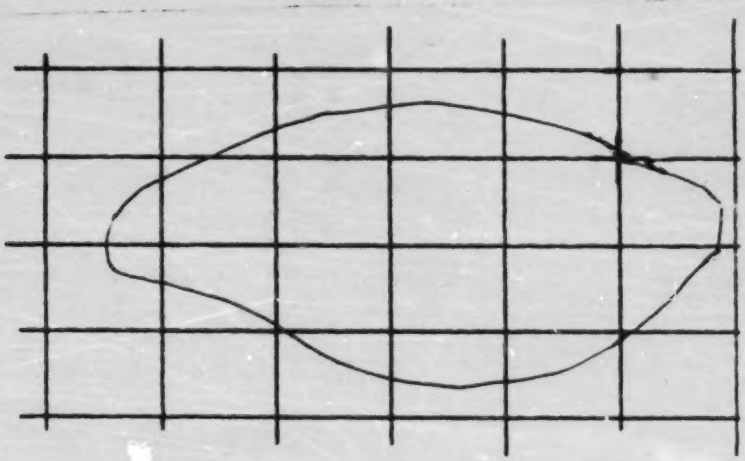


Illustrations of other types of grids can be found in SMP, Book B, pp. 41-43 and in Pearcy and Lewis, Experiments in Mathematics, Stage 3, pp. 12-13.

---

[1] SMSG - Mathematics for the Elementary School, Grade 5 (Part 3), Teachers Commentary, pp. 747-749.

a. To use a grid in measuring a region you superimpose the grid on the region as in the figure below. Study the grid, then answer the questions.



1. How many of the unit regions pictured are contained entirely in the given oval region?

_____

2. What does your answer to 1 tell you about the size of the area of the oval?

_____

_____

_____

_____

3. How many more units are partially covered by the oval?

_____

4. In light of 3, what would you say is a reasonable estimate of the oval region in terms of the unit of the grid?

_____

_____

_____

b. On a piece of graph paper, draw each of the following quadrilaterals connecting points in order:

H(2,0), E(0,2), R(2,6), B(6,2)

M(1,2), A(5,2), R(11,8), Y(7,8)

1. Using the graph paper unit, count the number of whole unit regions contained entirely within the quadrilateral regions.

   a. In quadrilateral HERB_____

   b. In quadrilateral MARY_____

2. What does your answer to 1 tell you about the size of the area of the quadrilateral region?

   a. In quadrilateral HERB_____

   b. In quadrilateral MARY_____

3. How many more units are partially covered by the quadrilateral?

   a. In quadrilateral HERB_____

   b. In quadrilateral MARY_____

4. What would you say is a reasonable estimate of the quadrilateral region in terms of the unit of the graph paper?

   a. In quadrilateral HERB_____

   b. In quadrilateral MARY_____

c. Complete Exercise 3 on page 44 in SMP, Book B.

1. The estimated area of the larger circle_____

2. The estimated area of the smaller circle is_____

3. What relationship did you find between the estimated areas of the two circles?

_____

_____

_____

_____

TURN TO THE NEXT PAGE AND CHECK YOUR RESPONSES.

Discussion of Responses:

a. 1. Six

2. The area of this region is at least six. The area is more than six.

3. 13 additional unit regions have some part covered.

4. The area of the region is somewhere between six units and nineteen units; a reasonable estimate might be 11 units.

b. 1. a. 11

b. 18

2. a. Area is greater than 11

b. Area is greater than 18

3. a. 14 b. 12

4. a. 18 b. 24

c. 1. Estimate: 50 cm^2

2. Estimate: 13 cm^2 or 12 cm^2

3. The area of the larger circle is 4 times the area of the smaller circle.

Have you completed Task 3.2 satisfactorily?

Yes - Proceed to Task 3.3

No - Is this your first try?

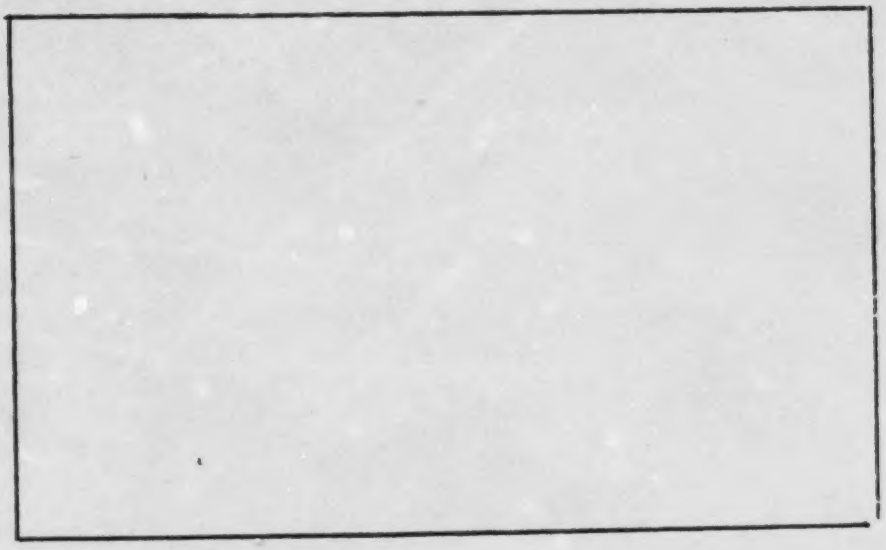
Yes - Read one of the suggested texts on page 60 and redo exercise.

No - See instructor.

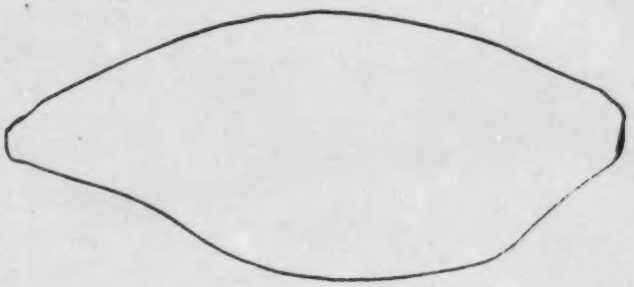
Task 3.3.

a. Construct a unit grid of one inch square below:

GRID: Unit--one-inch square



Trace the above grid. Superimpose it on the curved region below.



1. How many of the unit regions pictured are contained entirely in the given oval region?

2. What does your answer to 1 tell you about the size of the area of the oval?

3. Approximately how many more covering units can you verify by counting?

4. In light of 3, what would you say is a reasonable estimate of the area of the oval region in terms of the unit of the grid?

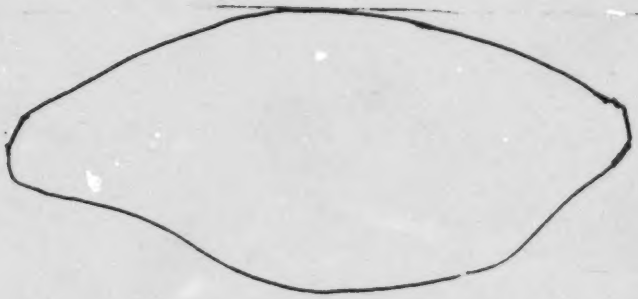
5. Compare your answer to 4 above with your answers to Task 3.2a part 4 . Why do they differ?

The unit used in this grid, namely, a square inch, is considered to be a **standard unit** of measure.

b. Construct a unit grid of one centimeter squares below:

GRID: Unit--one centimeter square

Trace the above grid. Superimpose it on the curved region below:



1. How many of the unit regions pictures are contained entirely in the given oval region?

2. What does your answer to 1 tell you about the size of the area of the oval?

3. Approximately how many more covering units can you verify by counting?

4. In light of 3, what would you say is a reasonable estimate of the area of the oval region in terms of the unit of the grid?

5. Compare your answer to 4 above with your answers to Task 3.2a, part 4. Why do they differ?

The unit used in this grid, namely, a square centimeter, is considered to be a standard unit of measure.

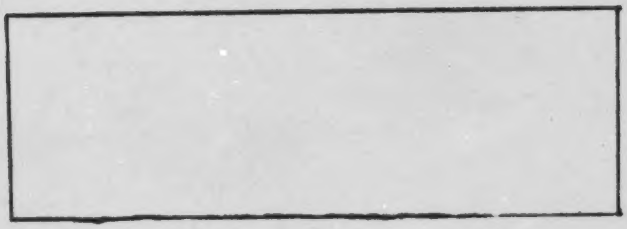
c. Trace a small section of both the one-inch square unit grid and the one-centimeter square grid. Superimpose the one-inch square grid in the one-centimeter square grid.

1. Which unit is larger? _____

2. Estimate the number of square centimeters that cover a square inch.

d.

8"



2'

Suppose you have a rectangle with length 2' and width 8". Could you find a suitable rectangular region which would fit into the given rectangle exactly sixteen times?

1. What might this unit be?_____

2. What name would you give it?_____

3. Would this be a good unit to use as a means of determining the area of a region?

_____

_____

_____

_____

_____

4. Justify your answer to part 3.

_____

_____

_____

_____

_____

5. Would you classify this unit as a standard unit of measure?

_____

_____

TURN TO THE NEXT PAGE AND CHECK YOUR RESPONSES.

Discussion of Responses:

a. 1. The answers will vary depending upon how you superimpose the grid.

2. Same answer as above.

3. Same answer as above.

4. Same answer as above.

5. Comparison will vary. The answers differ because of the size of the unit.

b. 1. The answers will vary depending upon how you superimpose the grid.

2. Same answer as above.

3. Same answer as above.

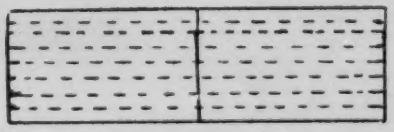
4. Same answer as above.

5. Comparison will vary. The answers differ because of the size of the unit.

c. 1. Square inch.

2. Estimate: 5 or 6 square centimeters.

d. 1. A unit one foot long and one inch wide or another possibility 2 feet long and $\frac{1}{3}$ inch wide. Can you suggest a third possibility?



8 inches

2 feet

2. The name of the unit would be an inch foot or a foot inch in the first case. One could name the unit in the next case a "Skinny." Have you a better name?

3. It would be a unit to determine the area of a region, but not a convenient one. A convenient one would have the same linear measurement for both length and width.

4. It would be difficult to make measurement comparison with such a unit.

5. No. It is a nonstandard unit of measure.

Have you completed Task 3.3 satisfactorily?

Yes - Proceed to Post-Assessment for Element III, page 61.

No - Is this your first try?

Yes - Read one of the suggested texts on page 60 and redo exercise.

No - See instructor.

Activity 2

From the following list of books, select three and study the pages indicated. When you have finished the reading, turn to the post-assessment for Element III on page 61 and follow the directions given there.

1. Hooten, Mahaffey, Elementary Mathematics Laboratory Experiences, pp. 75-77.

2. Pearcy and Lewis, Experiments in Mathematics, Stage 3, pp. 12-13.

3. SMP, Book B, pp. 41-46.

4. SMSG, Book 5, pp. 746-751.

Post-Assessment

In Task 1.1, page 19 you classified certain objects as plane regions. For each of the objects you classified as plane regions, list in the table below standard and nonstandard units which could be used to determine the area of the region.

a. TABLE III

Plane Region Objects	Name of Unit of Measurement	
	Standard	Nonstandard

b. Justify your classifications

_____

_____

_____

_____

_____

_____

_____

_____

Directions

1. Check your responses to this post-assessment on page 4 of the RESPONSE MANUAL.

2. a. Score of Your Responses to Table III.

   Number Correct_____

   Number Incorrect_____

   b. Your response to part b.

   Agrees? Yes_____ No_____

3. If you had more than five responses incorrect on Table III and answered No to Direction 2b above, read one of the suggested readings given on page 60 and redo the task. When you have redone the task satisfactorily, go on to Direction 4 and continue.

4. If you have less than five incorrect responses and answered Yes to Direction 2b above, go to page 10.

5. On your Decision Chart, place a check in column 3 to indicate that you have completed Element 3.

6. Now turn to Section II, read and complete the material on pages 185-188. Then complete Activity A which starts on page 189.

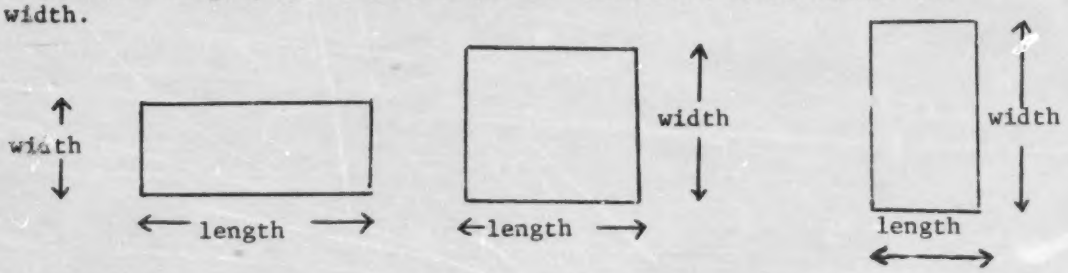
ELEMENT IV

Objective: Given a region, you will be able to determine the change in area as the dimensions are varied.

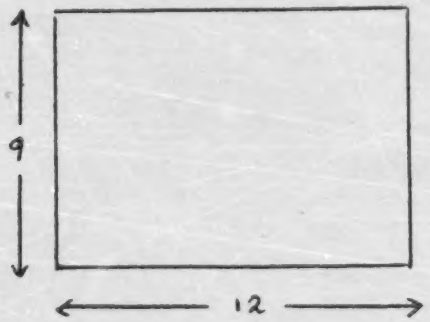
This element is designed to enable you to discover the effect of a change in dimensions on the size of an area. Like all the elements, it begins with a short introduction to the terms and concepts you will be using. After you have become familiar with these concepts, you can choose to complete either Activity 1--a series of tasks specially written for this module--or Activity 2--guided readings. When you have finished your chosen activity, turn to page 87 for the post-assessment for this element. Satisfactory completion of the post-assessment means that you have met the objective of this element and may go on to the next circled element in your Decision Sheet.

Background

Dimensions of a rectangle can be given many names or labels--length, width, depth, height, base, breadth. Although any two are appropriate, the horizontal measurement for this section will be called the length, and the vertical measurement will be called the width.

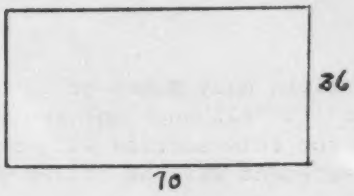


A figure given as 9 x 12 will mean that the first numeral refers to the width and the second numeral to the length.

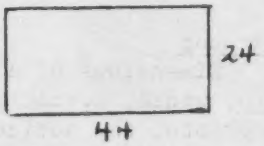


Many things in our daily life can be covered by something or show a surface area--a lawn by grass, a table by cloth, a floor by rugs. Because of this, many choices in life involve questions such as the following: If a large bath towel 36 x 70 costs $4.98 and a regular bath towel 24 x 44 costs $2.24, is the change in cost warranted by the change in size? From the cost change you would expect the large towel to be more than double in surface area. You could check this by making a diagram of the towels. To make a diagram, form a model of the smaller towel and find how many times it fits over the larger. (See diagrams below). Note that it may be necessary to cut the smaller towel in pieces to make it fit on the large towel. If it fits more than twice, then the change in price is worth it.

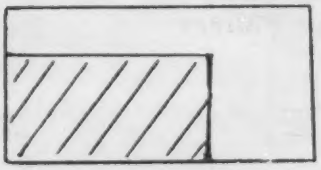
TOWEL I
36 x 70



TOWEL II
24 x 44

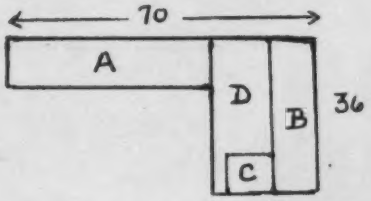


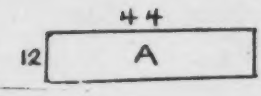
Place towel II on Towel I



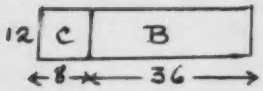
Note L-shaped region remaining uncovered.

You can see it will be necessary to cut the L-shaped region into pieces that may be put together to form a region equivalent to towel II.





Region A is half of towel II



Combination of regions B and C make up the other half of towel II

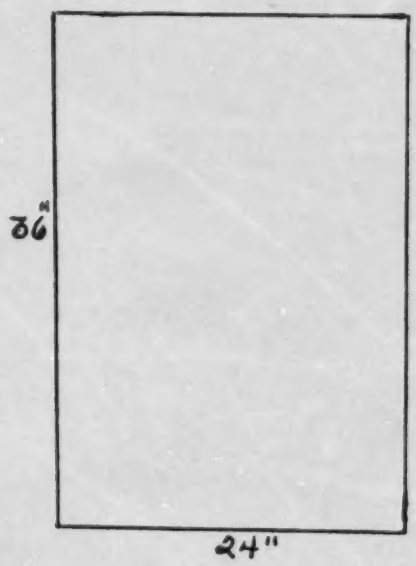
Thus it is clear that the L-shaped region is larger than towel II because you have region D left over.

Hence, it would appear that the change in cost is warranted by the change in size.

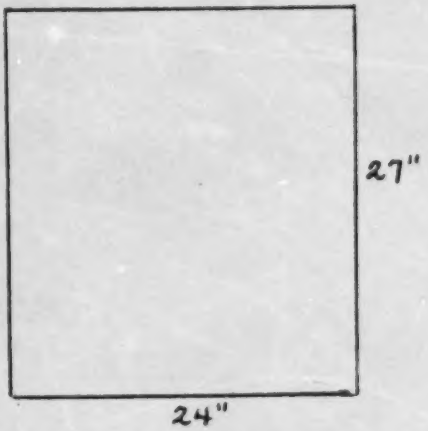
Here are two examples for you to try:

1. Show by diagram how many times a 12" x 18 " hand towel will fit over each of the following towels:

a.



b.



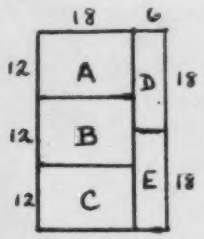
2. If a rug 9' x 12' costs $705.00, does the price $395.00 sound reasonable as a price for the same style rug which measures 6' x 8'?

Justify your answers (a diagram might be helpful).

CHECK YOUR RESPONSES ON PAGE 68.

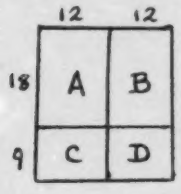
ANSWERS TO PRACTICE EXAMPLES ON PAGE 66 .

1. a.



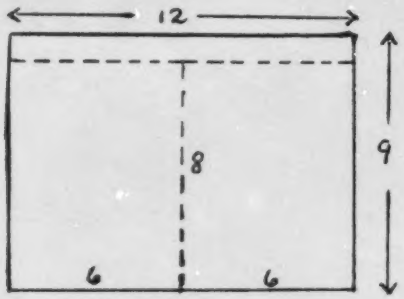
Regions A, B, C are each the same size as the 12 x 18 sized towels and regions D and E together make a region equivalent to a fourth 12 x 18 towel.

b.



Regions A and B are each the same size as the 12 x 18 towel and regions C and D together make a region equivalent to a third 12 x 18 towel.

2. The price seems reasonable since you can get two 6 x 8 rugs from the 9 x 12 rug and have a piece left over.



NOW TURN TO THE NEXT PAGE IF YOU HAVE CHOSEN ACTIVITY 1 AND TO PAGE 86 IF YOU HAVE CHOSEN ACTIVITY 2.

Activity 1

If you have chosen this activity you will be asked to do two tasks and check them yourself. You must do Task 4.1; you may choose either 4.2 or 4.3 for the second task. If you have any problems or feel that you need more guidance, ask your instructor's advice.

Before you start the tasks, see that you have the following materials on hand:

Scissors
Unlined paper
Graph paper
Dot paper
Geoboards - one 5 x 5, one 10 x 10 or 11 x 11
Centimeter graph paper
SMP, book D
SMP, book D Teacher's Guide

Task 4.1. You are to read SMP, Book D, pages 23-27, then do the exercises on pages 27-30 in the space below. (You may need extra paper for this.)

CHECK YOUR RESPONSES ON THE NEXT PAGE.

Discussion of Responses

Consult SMP, Book D, Teacher's Guide for answers to the exercises on pages 27-30.

Have you completed Task 4.1 satisfactorily?

Yes - Select Task 4.2 (page 73) or 4.3 (page 79) and complete it.

No - Is this your first try?

Yes - Read one of the suggested texts and redo exercise.

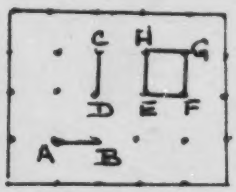
No - See instructor.

Task 4.2 Background

In order to do this task, make sure you understand the explanation below:

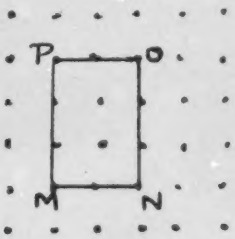
On the geoboard, make the following figures:

1.



The line segment AB represents a unit of base (length).
The line segment CD represents a unit of height (width).
The region enclosed by square EFGH is one unit of area.

Consider the following diagram:



Name the line segment representing the base (length) _____
Write the number of units in the base. _____
Name the line segment representing the height (width). _____
Write the number of units in the height. _____
Write the number of units in area MNOP. _____

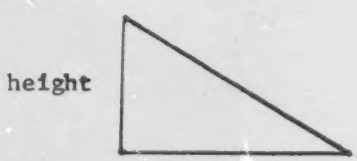
Did you write PO or MN for the base? It has 2 units.

Did you write PM or ON for the height? It has 3 units.

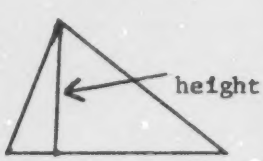
Did you find that MNOP has an area of 6 units?

When you talk about the dimensions of a rectangle, you usually use the terms <u>length</u> and <u>width</u> but when you talk about triangles you refer to their <u>bases</u> and <u>heights</u>.

length



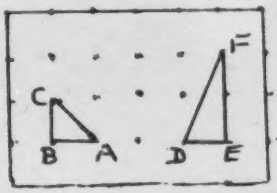
base



base

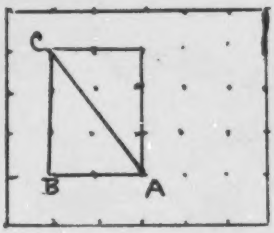
Now make the following figures:

2.



Triangle ABC is a right triangle with the right angle at B.
Triangle DEF is a right triangle with the right angle at E.

3.



To find the area enclosed by a right triangle, form a rectangle using the same base and height. Determine the number of units of area in the rectangle. How does the area enclosed by the triangle compare with the area enclosed by the rectangle? In the case pictured the rectangle encloses an area of 6 units and the triangle ABC encloses an area of 3 units.

As a check, make a right triangle with two units on the base and two units in height on your geoboard. Find the area. If you agree that the area enclosed by the triangle is 2 square units, you are right.

NOW TURN TO THE NEXT PAGE FOR THE TASK ITSELF.

Task 4.2

a. On your geoboard, with a rubber band make a right triangle one unit on the base and three units in height in the lower right-hand corner.

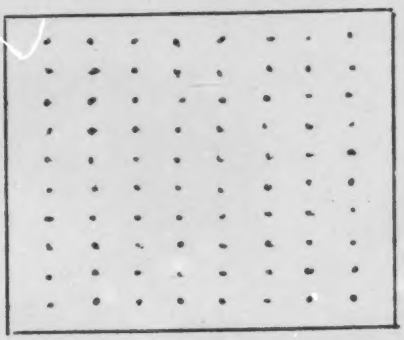
Take another rubber band and make a triangle which has the same base but whose height is one unit more. You do this by placing the rubber band over the original base and extending the height by one unit.

You now have two triangles on your geoboard which have the same base but different heights.

With a third rubber band, make a right triangle one unit on the base and two units in height in the lower left-hand corner of the geoboard.

Place a fourth rubber band on the same base and make the height one unit higher.

Record the shapes on the dot paper below.



Record the base, height and area on Table 4.2a

Table 4.2a

	Base	Height	Area
Right Corner			
Tri I	____	_____	____
Tri II	____	_____	____
Left Corner			
Tri III	____	_____	____
Tri IV	____	_____	____

Compare the sets. Describe in a statement how the change in height effected the area.

_____

_____

_____

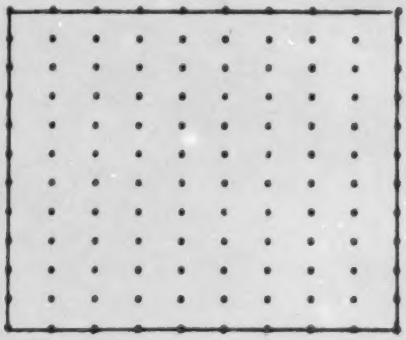
_____

b. Clear the geoboard. Make a right triangle two units on the base and one on the height in the left-hand bottom corner.

Use another rubber band and make a right triangle in that corner with the same base but double the height.

In the right-hand bottom corner, make a right triangle two units on the base and two on the height.

Make a second triangle on the same base but double the height. Record the shapes on dot paper.



Record the base, height, and area of each on Table 4.2b

Table 4.2b

	Base	Height	Area
Left Corner			
Tri V	____	_____	____
Tri VI	____	_____	____
Right Corner			
Tri VII	____	_____	____
Tri VIII	____	_____	____

Study the entries in the table. Compare the changes of the two triangles on the left corner with the changes of the two triangles on the right corner.

Describe the changes in the sides and their effects on the area.

_____

_____

_____

_____

_____

_____

_____

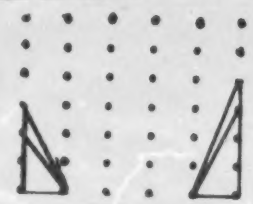
TURN TO THE NEXT PAGE AND CHECK YOUR RESPONSES.

Discussion of Responses

Table 4.2a

	Base	Height	Area
Right Corner	1	3	1½
	1	4	2
Left Corner	1	2	1
	1	3	1½

Does your geoboard look like this?



If so, continue and check Table 4.2a. If not, see where you went wrong and correct it and redo the Table 4.2a.

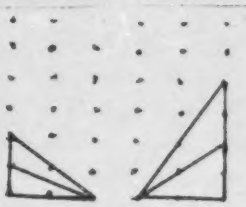
1. A possible response is--When the height increased by one height unit and base remained one unit, the area increased by ½ area unit.

or

Original Area = ½bh area unit. Since b = 1, original area = ½h area units.

New Area = ½b (h + 1) Since b = 1, new area = ½h + ½ area units.

b. Does your geoboard look like this?



If so, continue and check Table 4.2b. If not, see where you went wrong, correct it and redo Table 4.2b.

If you are not sure why the board should look like this and you had trouble with a, go to the laboratory instructor for further help. Then continue.

Table 4.2b

	Base	Height	Area
Left Corner	2	1	1
	2	2	2
Right Corner	2	2	2
	2	4	4

1. A possible response is - When the base unit remains the same and the height unit is doubled, the area is doubled.

or

Original Area = $\frac{1}{2}bh$
New Area = $\frac{1}{2}b\ (2h) - bh$

Have you completed Task 4.2 satisfactorily?

Yes - Is this your second task for Activity 4?

Yes - Proceed to Post-Assessment for Task 4.2 , page 87 .

No - Select one more task in Activity 4 and complete it.

No - Is this your first try?

Yes - Read one of the selected texts and redo exercise.

No - Do another task in Activity ' or see instructor.

Task 4.3. Use centimeter graph paper for this task.

a. Cut out four rectangular regions 4 cm. in width by 8 cm. in length.
Paste one in the space provided below.
Record the length, width, and area in the table.

Take a second region. Fold it in half vertically.
Cut it in two. Paste one part in the area provided below.
Fold the other in half vertically again. Cut along the fold.
Paste one piece in the space provided below. Continue the process once more.

Fill in the data in the table.

Paste one whole rectangular region below.

First Cut Second Cut Third Cut

Table 4.3a

	Length	Width	Area
Whole	_____	_____	____
Vertical Cut			
First Paste	_____	_____	____
Second Paste	_____	_____	____
Third Paste	_____	_____	____

Look at the Table and shapes. Study the data. Do you observe a pattern? State it.

_____

_____

_____

_____

b. Take the third region 4 cm. by 8 cm. Fold this one in half horizontally. Cut along the fold. Paste one piece in the space below.

Record the length, width, and the area in Table 4.3b.

Continue the folding and pasting twice more.
Record the length, width, and area for each cut in the table.

Paste Picture of

First Cut

Second Cut

Third Cut

Table 4.3b

	Length	Width	Area
Whole	_____	_____	____
Horizontal Cut			
First Paste	_____	_____	____
Second Paste	_____	_____	____
Third Paste	_____	_____	____

Look at the table and the shapes. Study the data. State the pattern that you observed from the effect of cutting the area.

_____

_____

_____

_____

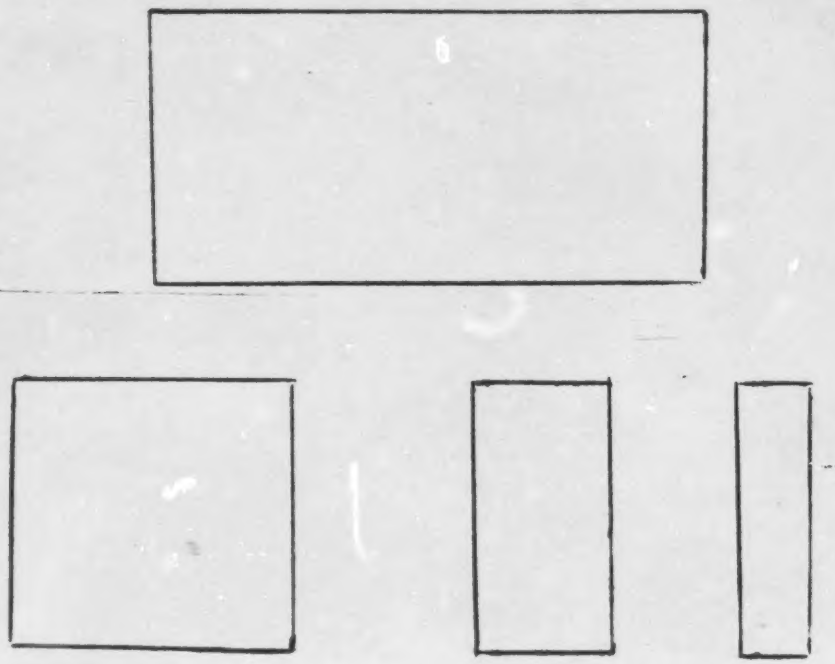
_____

NOW LOOK AT THE ANSWERS ON THE NEXT PAGE.

Discussion of Responses:

Task 4.3

a. Does your paste picture look like the one below?



Yes - Check your responses in Table 4.3a on the next page.

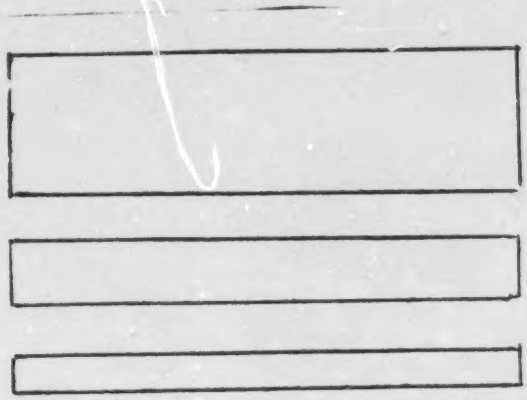
No - Cut another set of rectangles and do again.

Table 4.3a

	Length	Width	Area
Whole	8	4	32
Vertical Cut			
First Paste	4	4	16
Second Paste	2	4	8
Third Paste	1	4	4

The pattern is the following: as the length is halved and the width remains the same the area is halved.

b. Does your paste picture look like the one below?



Yes - Check your responses in Table 4.3b on the next page.

No - Cut and fold again.

Table 4.3b

	Length	Width	Area
Whole	8	4	32
Horizontal Cut			
First Paste	8	2	16
Second Paste	8	1	8
Third Paste	8	½	4

The pattern is the following: as the width is halved and the length remains the same the area is halved.

Have you completed Task 4.3 satisfactorily?

Yes - Is this your second task for Activity 4?

Yes - Proceed to Post-Assessment for Task 4.3 on page 90.

No - Select one more task in Activity 4 and complete it.

No - Is this your first try?

Yes - Read one of the suggested texts on page 86 and redo exercise.

No - Do another task in Activity 4 or see instructor.

Activity 2

From the following list of readings, select three and study the indicated pages. When you have finished the reading, turn to the post-assessment for Element IV on page 87 and follow the directions given there.

1. Biggs, Freedom to Learn, pp. 9-13.

2. Copeland, How Children Learn Mathematics, pp. 231-238.

3. Lovell, The Growth of Understanding Mathematics, pp. 107-114.

4. Copeland, Mathematics and the Elementary Teacher, pp. 50-68.

5. SMP, Book D, pp. 23-30.

Post-Assessment

Select the post-assessment task that corresponds to the task that you have completed for the element. (There is no post-assessment for 4.1).

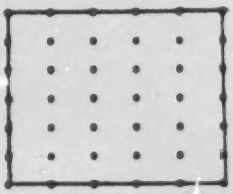
Post-Assessment for Task 4.2

1. Clear the geoboard. Make a right triangle two units on the base and one on the height in the lower left corner.

   Make a second right triangle on the board with double the base and double the height.

   Using rubber bands or a cut out triangle, see how many times the smaller triangle will fit into the larger triangle.

   Record the shapes on the dot diagram below:

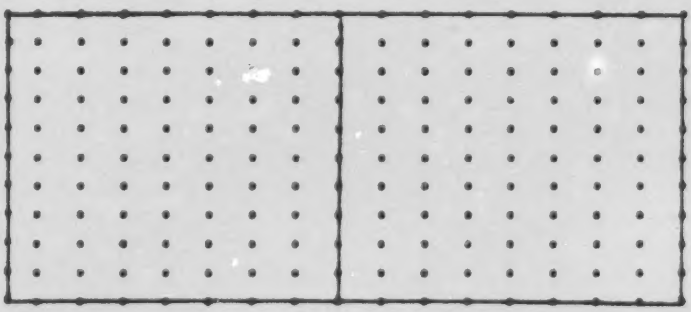


   Record the data on the base, height, and area of each in Table 4.2c.

2. Clear the geoboard. You will need two geoboards 5 x 5 for this activity or a geoboard 10 x 10 or 11 x 11.

   Make a right triangle three units on the base and two units for the height. Make a second right triangle on the other board. Double the base and double the height.

   Record the shapes on the dot diagram below:



   Record the data on the base, height, and area on each in Table 4.2c.

3. Clear the geoboard. Make a right triangle of your own. Make a second right triangle over it which follows the same pattern.

Record the shapes on dot paper below and data in Table 4.2c.

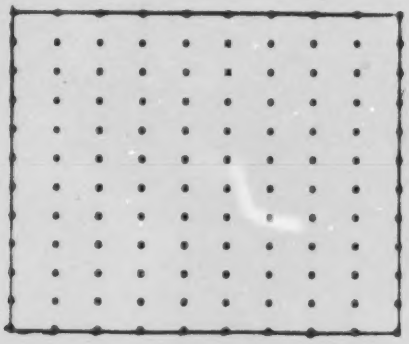


Table 4.2c

Triangle	Base	Height	Area
IX	____	_____	____
X	____	_____	____
XI	____	_____	____
XII	____	_____	____
XIII	____	_____	____
XIV	____	_____	____

4. Describe the changes in the base and height with their effect on the area.

_____

_____

_____

_____

Directions

1. Check your responses on pages 5-6 of the RESPONSE MANUAL.

2. Are your responses to Task 4.2 satisfactory?

   Yes_____

   No_____

3. If you have checked NO to the above question read one of the suggested readings on page 86 and redo the post-assessment task. Then go to Direction 4 and continue.

4. If you haved checked YES to Question 2, go back to page 10.

5. On your Decision Chart, place a check in column 3 to indicate you have completed the original activity.

6. Go to the next circled item. Follow the directions given for that item.

Post-Assessment for Task 4.3

Take the fourth rectangular shape. Fold it in half vertically and in half horizontally. Cut along the folds. Paste one piece in your notebook. Repeat this process again.

Record the data in Table 4.3c below:

Table 4.3c

	Length	Width	Area
Whole	_____	_____	____
Double Fold			
First Cut	_____	_____	____
Second Cut	_____	_____	____

Describe the relationship of change in sides to change in area in words and in an open sentence.

_____

_____

_____

_____

Directions

1. Check your responses on page 7 of the RESPONSE MANUAL.

2. Are your responses to Task 4.3 satisfactory?

   Yes_____

   No_____

3. If you have checked NO to the above question, read one of the suggested readings page 86 and redo the post-assessment task. Then go to Direction 4 and continue.

4. If you have checked YES to Question 2, go back to page 10.

5. On your Decision Chart, place a check in column 3 to indicate that you have completed the original activity.

6. Go to the next circled item. Follow the directions given for that item.

ELEMENT V

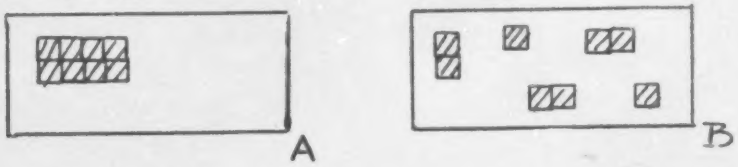
Objective: Given a series of experiments, you will be able to identify those that probe conservation of area.

Given a set of geometric figures, you will be able to restructure these figures to demonstrate equivalence of area.

In order to meet the above objectives you must successfully complete one activity and a post-assessment. You may choose the specially written materials and task in Activity 1 or the books suggested in Activity 2. In either case, you will complete the post-assessment to determine whether you have met the objectives. Before you begin, read the Background section that starts below.

Background

Ask a child, "Is more surface area covered by tiles (shaded sections) on board A or board B?"



The answer the child gives depends on his level of understanding of the concept of conservation. If a child has developed the concept of conservation,

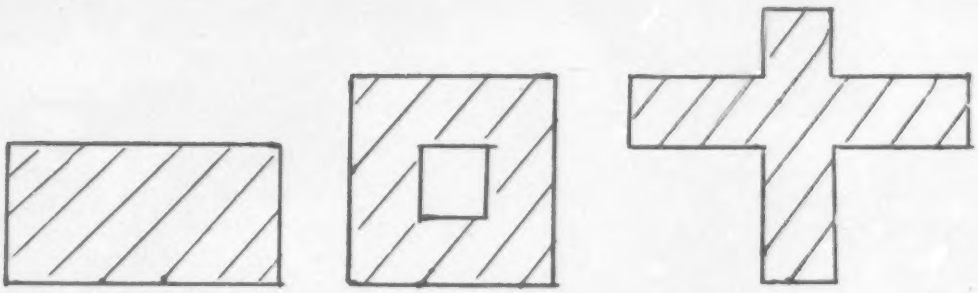
1. he will explain that the surface covered in A and B is the same but the tiles in B are more spread out, and -

2. he will also be able to say that if you took all the blocks in B and put them together, they would fit the shaded section of A exactly.

Thus for a full understanding of conservation the child must see first, that placing a unit measure onto another region covers the same space each time as indicated in 1 above and second, that when subdividing a surface into unit measures the whole is equal to the sum of its parts as indicated in 2 above.

Here is a problem for you to try. Compare the sizes of the following, using a square centimeter cut from cardboard as a pattern. Each time you place it down on a figure, trace its boundary in order to record its positions.

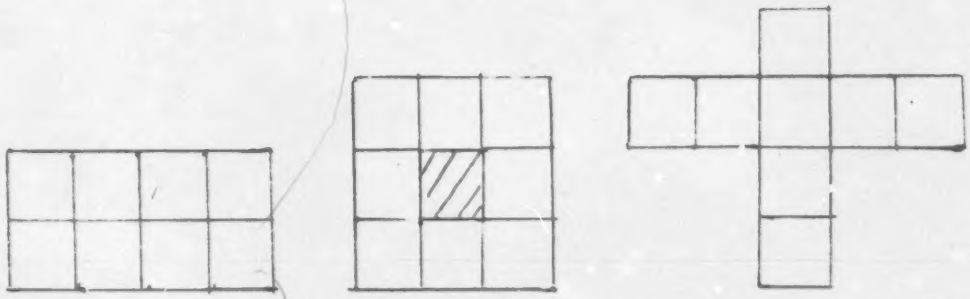
1 sq. cm.



No doubt you have noticed that 8 square centimeter units cover each of the above.

Did your result look like this?

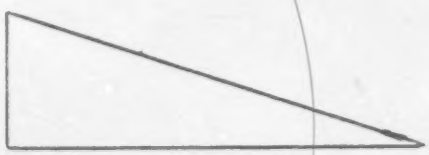
TURN TO NEXT PAGE.



It is clear you possess the concept of conservation of area since you have confidence that each time you use the unit it covers the same space and that the sum of 8 square centimeters in each region describes the measure of the whole region.

You were sure that although the three regions looked different each was covered by the same number of equivalent parts. When two regions are covered by the same number of equivalent parts, they are said to be <u>equal in area</u>.

Here is a problem for you to try. Trace the tile below and cut out four tiles like it.



Using all four tiles can you create:

1. a rhombus?

2. a triangle?

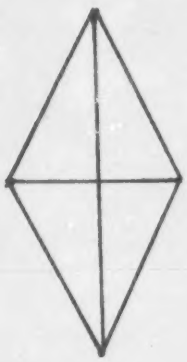
3. a square with a square whole in the middle?

What did your results look like?

Do they agree with those presented in the next page?

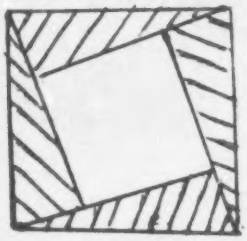
(Don't look until you have tried.)

(1)



(2)

(3)



Is the rhombus equal in area to the triangle?

Is the area of the rhombus the same as the area of the square with square hole in the middle?

It should be apparent to you that all three figures are equal in area since they are made up of the same number of equivalent parts.

NOW TURN TO THE NEXT PAGE, IF YOU CHOSE ACTIVITY 1 AND TO PAGE 113 IF YOU CHOSE ACTIVITY 2.

Activity 1

1. Do either Task 5.1 or 5.2.
   When you have completed the task, check the appropriate box.

Task	Completed
5.1	
5.2	

2. Do any TWO tasks from among Tasks 5.3, 5.4, and 5.5.
   When you have completed each task, check the appropriate box.

Task	Completed
5.3	
5.4	
5.5	

   Have you completed TWO tasks?

3. Do Post-assessment Tasks I and II. When you have completed the tasks check the appropriate boxes.

Task	Completed
I	
II	

4. You will need the following materials to help you complete the activities in this activity:

Scissors and rubber bands
Oaktag
Unlined paper, graph paper, dot paper
Scotch tape
Ruler (both centimeter and inch rulers)
Geoboards
Tangram pieces
Tangram activity cards
Tiles - right triangle and equilateral triangles

Films:

a. Dance Squared
b. Piaget on Conservation of Area

Task 5.1. The purpose of this activity is to learn the Piagetian tests for conservation of area and interpretation of the results. Work on any one of the following:

a. Watch Piaget's film," Conservation."
Prepare the test or tests used to determine the child's concept of conservation of area.

List the answers different children give that tell what level they are at:

b. Read Lovell, The Growth of Understanding in Mathematics, pages 107 to 114. Prepare the materials of the tests for conservation. Write the questions and steps you will take in giving the tests. State how you know what level the child is at.

c. Read Nuffield's Shape and Size, Book 2, pages 61-63. Prepare the postcard and cow tests. List the responses the child will give at each level.

_____

_____

_____

_____

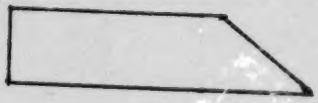
CHECK YOUR RESPONSES ON PAGE 103 AND RETURN TO DIRECTIONS ON PAGE 96.

c. 2. Compare the sizes of the figures a, b, c by filling them with the following shape tile:

Filling Tile

(a)

(b)



(c)

c. 3. How do these experiences relate to conservation of area?

Task 5.2. The purpose of this activity is to work with various activities that give experience with invariance of area.

Complete three of the following activities:

a. Watch the film "Dance Square."
Explain how this film gives experience in conservation of area.

b. Read Nuffield Shape and Size, Book 2, pages 63-67. List three different strategies used to give the experience of invariance of area.

c. 1. Using an equilateral triangle tile pattern, put exactly 16 such tiles together to form:

a. a triangle

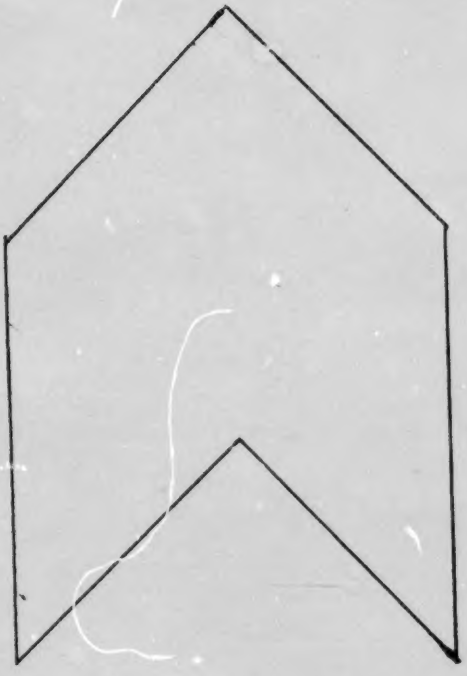
b. a parallelogram

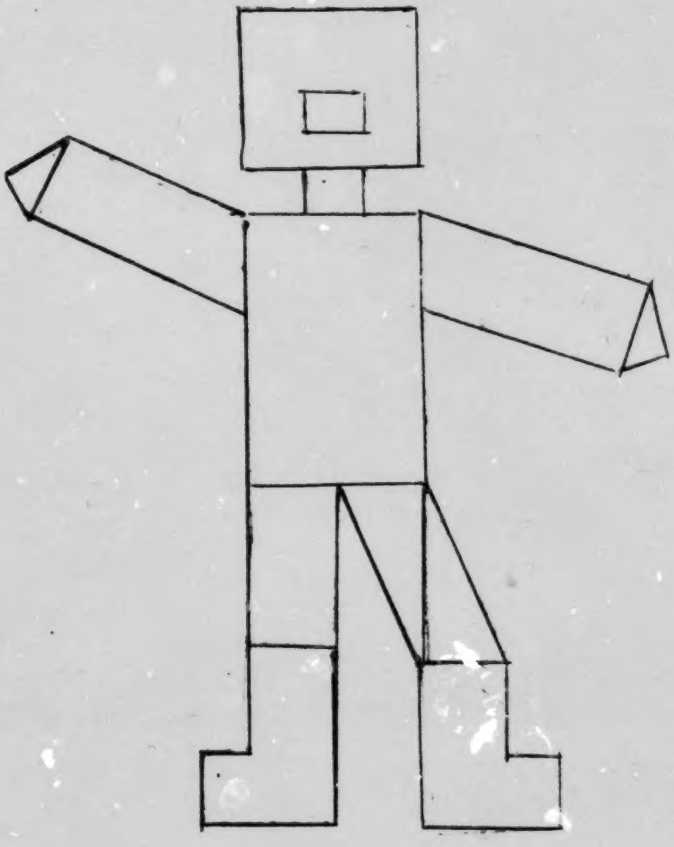
c. a trapezoid

Record your results.
What are your conclusions concerning the areas of these three figures?

Justify.

d. Look at the two figures below. Which covers more surface - the astronaut or the rocket? Prove the regions are equivalent. Trace the astronaut, cut it in pieces and try to fit all the pieces into the rocket.





How does this activity relate to the conservation of area?

TURN TO PAGE 104 AND CHECK YOUR RESPONSES.

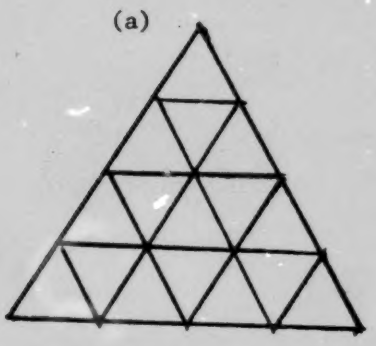
Discussion of Responses

Task 5.1. Follow reading material. Be sure you understand what to say and what to listen for.

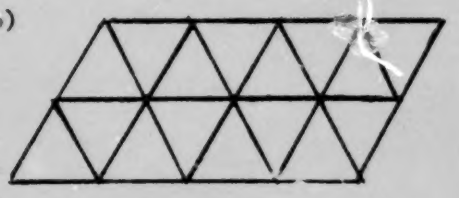
Task 5.2.

c. 1.

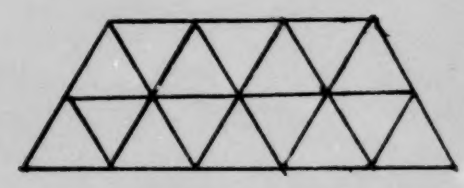
(a)



(b)

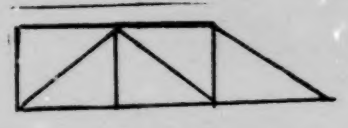


(c)



The three figures are equal in area since they are composed of the same number of equivalent parts.

c. 2. All these shapes are equivalent in area; each contains 5 congruent triangular regions.

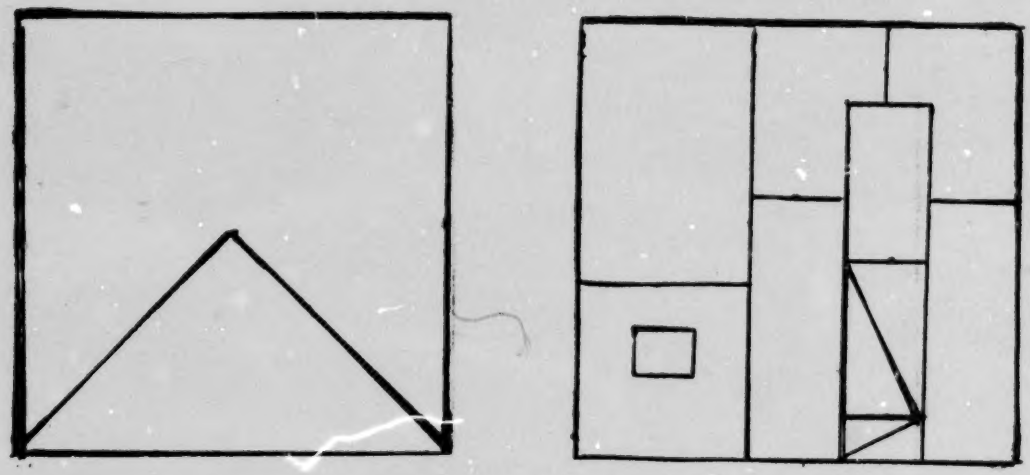


c. 3. Each uses the two criteria. The whole is filled by like parts. Although the unit has moved it covers the same surface area.

Discussion of Responses (cont'd.)

Task 5.2 (cont'd.)

d. The astronaut pieces fit into the square as do the rocket pieces.



The ability of two apparently different shapes having equivalent areas.

TURN BACK TO PAGE 96 AND FOLLOW REMAINING DIRECTIONS.

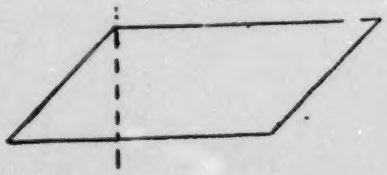
Task 5.3.

a. Cut out two paper parallelogram regions of the same size. Take one of the regions and make a vertical crease as indicated in the adjoining diagram. Cut along the crease to form two trapezoids.

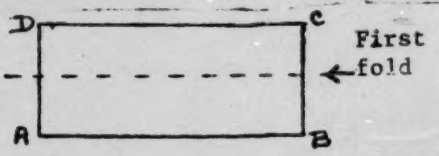
Can you reassemble the 2 trapezoids to form a rectangle?

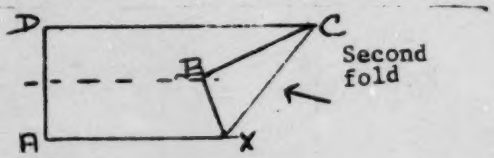
Take the other paper parallelogram region and make a crease as indicated in the adjoining diagram. Cut along the crease to form a right triangle and a trapezoid.

Can you reassemble the 2 pieces to form a rectangle?



b. Cut out a paper rectangular region the length of whose base and height are the same as the parallelogram in part (a).
Make 2 folds in the region as follows:
First fold the rectangular region in half horizontally; then unfold.
Now fold corner B up to the first fold and crease to make a triangle BCX. Cut along this crease to form a triangle (BCX) and a trapezoid.





Can you reassemble these two pieces to form a parallelogram?

From the experiments in a and b above, what conclusions concerning areas have you reached?

c. 1. All the faces of a rectangular solid are rectangles. Each of the Cuisenaire rods is an example of a rectangular solid. If a "train" is made of two orange rods and one red rod placed end to end, what is the total surface area of the "train"?

2. The white rod is one cubic centimeter. Why is the surface area of the "train" not equal to 22 cubic centimeters?

3. How does the measure of the surface area of a 4 centimeter cube compare to that of a rectangular solid with dimensions 2 cm. by 4 cm. by 8 cm.?

Justify your conclusion:

4. How do the volumes of the two solids compare?

TURN TO THE NEXT PAGE AND CHECK YOUR RESPONSES.

Discussion of Responses

Task 5.3

a. Yes.
Yes.

b. Yes.
Possible conclusions:

A given figure can be divided up in different ways. In each instance when the parts are reassembled, the area of the new figure is equivalent to the area of the original figure.

c. 1. One face of a white rod is a square centimeter. Using this as the area measuring unit, the total surface area of the "train" is 90 square centimeters.

(For further clarification refer to Task 3.2)

2. 22 cubic centimeters represents the VOLUME of the "train" not the surface area. That is, 22 white rods placed end to end would make up an equivalent "train."

3. Surface area of cube = 96 square centimeters.
Surface area of rectangular solid = 112 square centimeters.

Hence, surface area of rectangular solid is greater than surface area of cube.

4. Volumes of two solids are equivalent since both contain 64 cubic centimeters.

Have you completed Task 5.3 satisfactorily?

Yes - Return to page 96 and follow remaining directions.

No - Is this your first try?

Yes - Read one of suggested texts on page 113 and redo exercise.

No - See instructor.

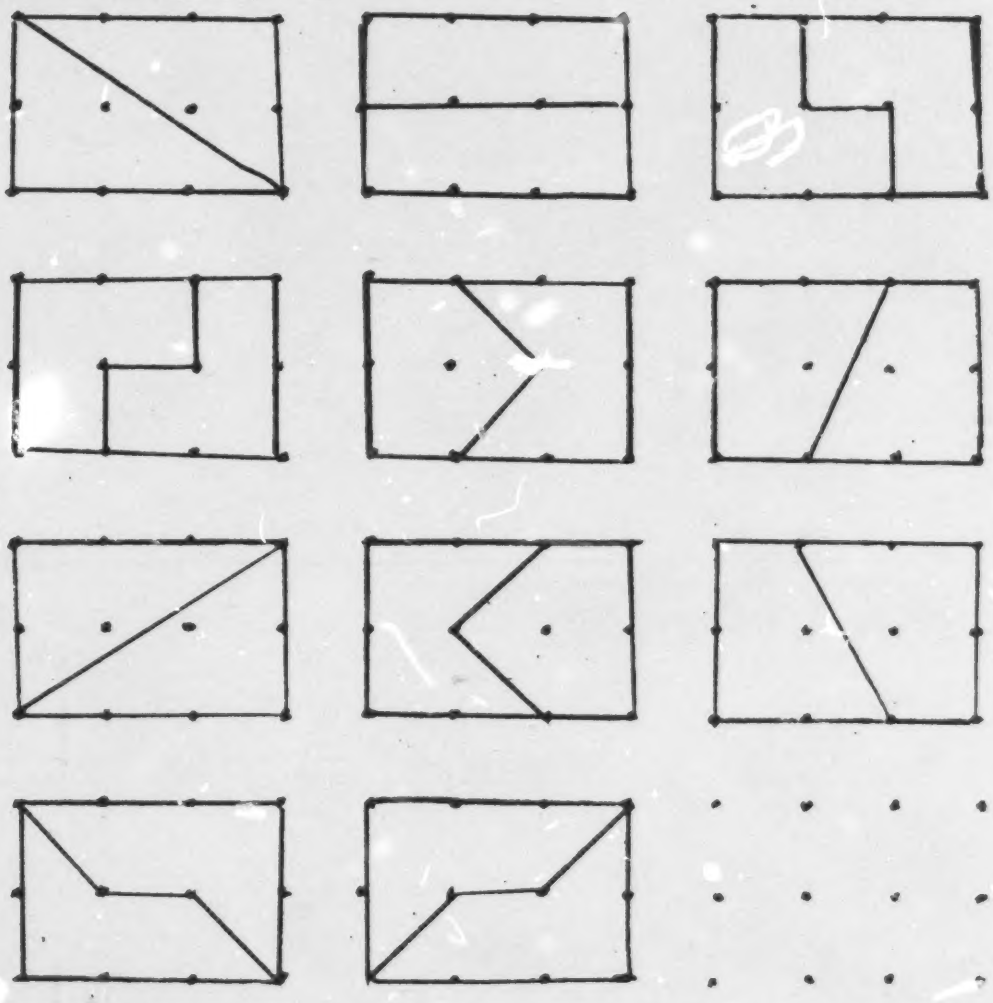
Task 5.4. On a geoboard construct a rectangle with length of 3 units and width of 2 units. Using one rubberband divide the rectangular region into two equivalent regions. Do it in at least 10 different ways. Record all your results on dot paper. (next page)

CHECK YOUR RESULTS ON PAGE 110.

Record results for Task 5.4.

Discussion of Responses

Task 5.4. Some possible constructions:



Have you completed Task 5.4 satisfactorily?

Yes - Return to page 96 and follow remaining directions.

No - Is this your first try?

Yes - Read one of suggested texts on page 113 and redo the exercise.

No - See instructor.

Task 5.5. Do one of the following:

a. Work through the complete set of Tangram activity cards and read the Tangram Teacher's Guide.

b. Complete the activities outlines on pages 149-152 and 159-169 of Fitzgerald, et al, Laboratory Manual for Elementary Mathematics, Second Edition.

c. Work through the Tangram activities on pages 83-92 of Hooten - Mahaffey, Elementary Mathematics Laboratory Experiences.

d. Work through all the experiments in Math Experiments with the Tangram.

TURN TO NEXT PAGE AND CHECK RESULTS.

Discussion of Your Responses

Task 5.5. See answer keys for individual books.

Have you completed Task 5.5 satisfactorily?

Yes - Return to page 96 and follow remaining directions.

No - Is this your first try?

Yes - Read one of suggested texts on page 113 and redo the exercise.

No - See instructor.

Activity 2

Read three of the books below and then turn to page 114 for the post-assessment tasks.

1. Copeland, How Children Learn Mathematics, pp. 229-243.

2. Copeland, Mathematics and the Elementary Teacher, pp. 50-68, pp. 149-150.

3. Elementary Science Study, Tangrams, Teacher's Guide.

4. Fitzgerald, et al., Laboratory Manual for Elementary Mathematics, Second Edition, pp. 149-152, pp. 159-169.

5. Hooten-Mahaffey, Elementary Mathematics Laboratory Experiences, pp. 83-93.

6. It's a Tangram World, Jenkins-McLean.

7. Lovell, The Growth of Understanding in Mathematics, pp. 107-114.

8. Nuffield Series, Shape and Size, Book 3, pp. 61-77.

9. Pearcy-Lewis, Experiments in Mathematics, Stage 1, pp. 6-9.

10. Piagetian, Cognitive-Developmental Research and Mathematics Education, pp. 189-199.

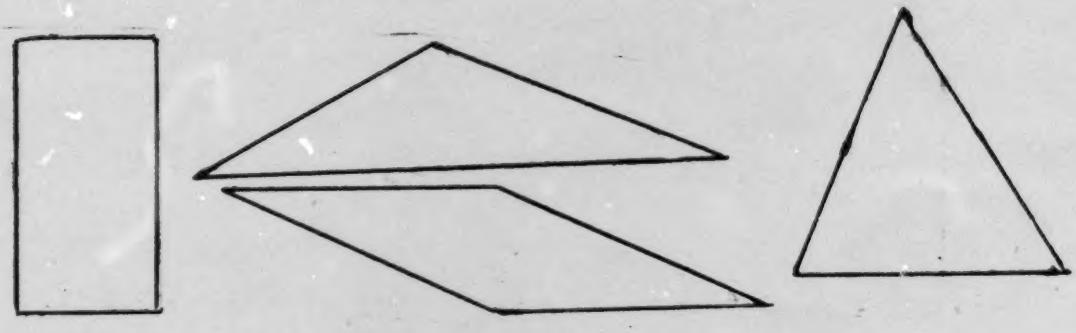
11. Seymour, Tangramath.

12. SMP, Book B, pp. 36-40, 50.

13. Tangram-Math Experiments, Midwest Publications.

Post-Assessment Task I

1. Cut out four rectangular shapes 4 cm. by 2 cm. Cut each along one diagonal to form two triangles. Place sets of two triangles together in different ways to form the following figures. Paste the triangles in each.

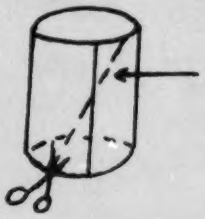


1 sq. cm.

Using 8 square cm. pieces, place them in each figure. Is each figure completely filled? Discuss your answers. (You may cut some of the square regions in half.)

2. Cut out three rectangular shapes three inches wide and five inches long. Roll the second so that the 3" edges meet. Scotch tape them. Roll the third so that the 5" edges meet. Scotch tape them. Take the formed cylinders and cut them obliquely. (see diagram)

Flatten out each figure.



oblique cut

Diagram each figure, the rectangular shape and those cut and flattened, using 1 cm. to equal 1".

3. What is true about all the figures when you compare them with 15 inch squares?

_____

_____

4. How does this activity relate to the conservation of area?

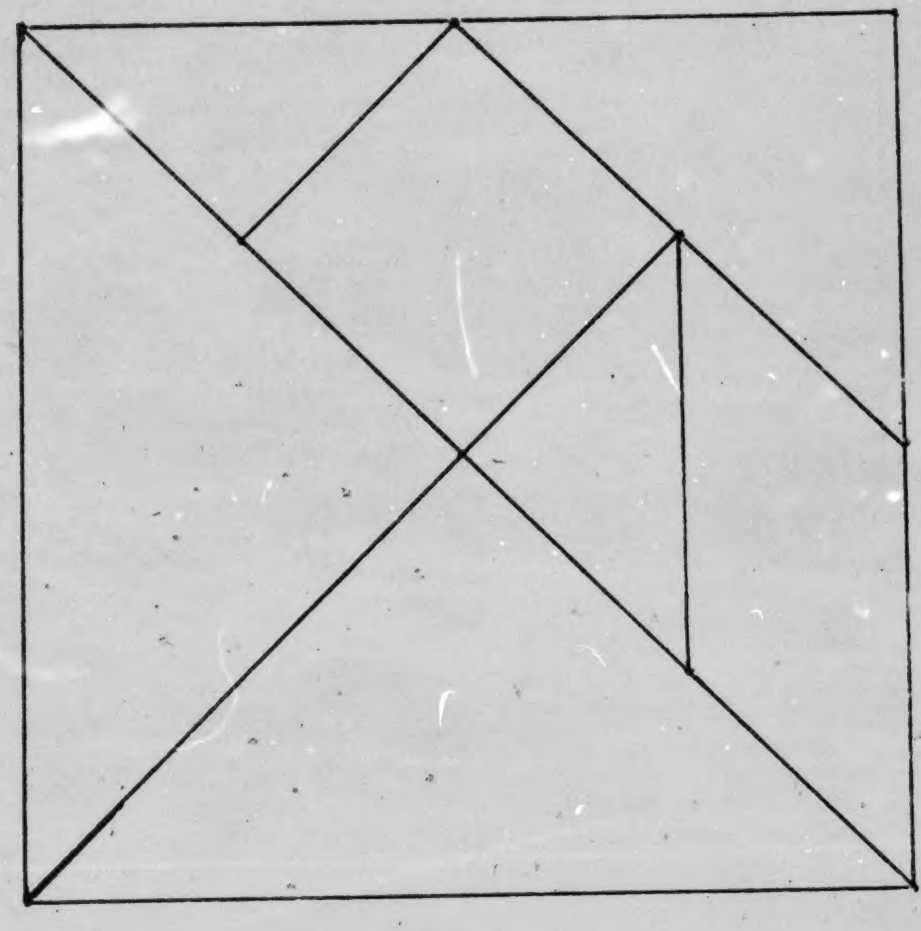
_____

_____

_____

TURN TO THE NEXT PAGE AND BEGIN POST-ASSESSMENT TASK II.

Post-Assessment Task II



a. Trace the above pattern on unlined paper. Cut out the 7 pieces of the puzzle.

TURN TO NEXT PAGE AND CONTINUE.

b. Using all 7 pieces each time, try to fit the pieces together to form:

1. a triangle

2. a rectangle (which is not a square)

3. a parallelogram (which is not a rectangle)

4. several trapezoids

5. a pentagon

6. several hexagons

7. a "cat"

Is there more than one possibility?

_____

Record five of your results.

c. What conclusions have you reached concerning the areas of all the figures formed in b above:

_____

_____

Directions

1. Check your responses to the Post-Assessment Tasks for Element 5 on pages 8-11 of the Response Manual.

2. Your responses to Task I

   Are they satisfactory? Yes_____

   No _____

   Your responses to Task II

   Are they satisfactory? Yes_____

   No _____

3. If you have checked No to either of the above questions, read one of the suggested readings and redo the post-assessment task(s).

4. If you have checked Yes to both of the questions in (2) above, go back to page 10.

5. On your Decision Chart, place a check in column 3 to indicate you have completed Activity 5.

6. Now turn to Section II and complete Activity B on pages 192-194.

ELEMENT VI

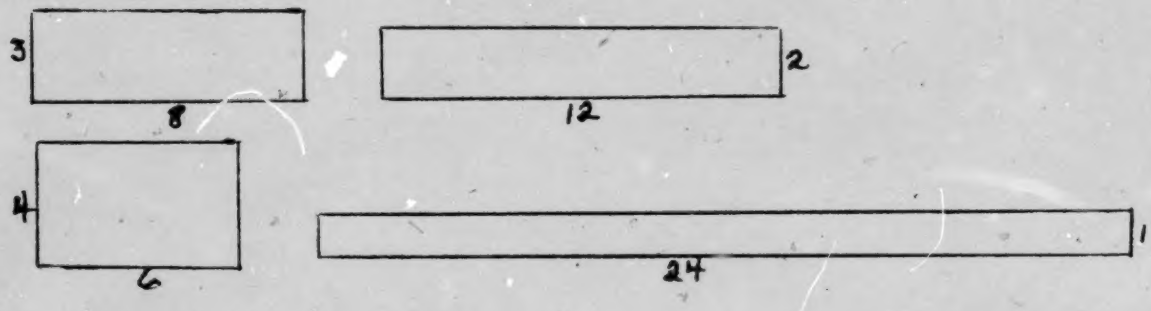
Objective: Using a set of materials, you will be able to demonstrate the following:

1. how to order them so that they lead to a discoverable pattern; and
2. how to derive the formulas for the areas of selected geometric figures.

Like all the other elements, this element consists of two activities and a post-assessment. Choose the activity you prefer--Activity 1, specially written materials; Activity 2, outside texts--complete it, then go to the post-assessment. When you achieve a satisfactory score on the post-assessment, you have met the objective of this element. But first, read the Background material below.

Background

Example: Take 24 congruent square tiles and make as many different rectangles as you can using all the tiles each time. Possible solutions would be



Record in the table below the length, width, perimeter and area of each of the above figures.

Length	Width	Perimeter	Area (number of tiles)
8	3	22	24
6	4	20	24
12	2	28	24
24	1	50	24

Do you notice any patterns from the above data concerning:
1. the perimeter of a rectangle?
2. the area of a rectangle?

From the patterns you noted, predict the perimeter and area enclosed by a rectangle with sides of length 4 units and 2.5 units.

perimeter = _____ area = _____

If you predicted the perimeter was 13 units and the area was 10 square units, you are ready to generalize your discoveries and predict the perimeter and area enclosed by a rectangle with sides of length a units and b units.

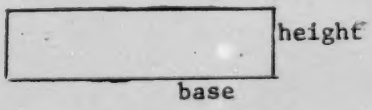
perimeter = _____ area = _____

How do your predictions compare with the following?

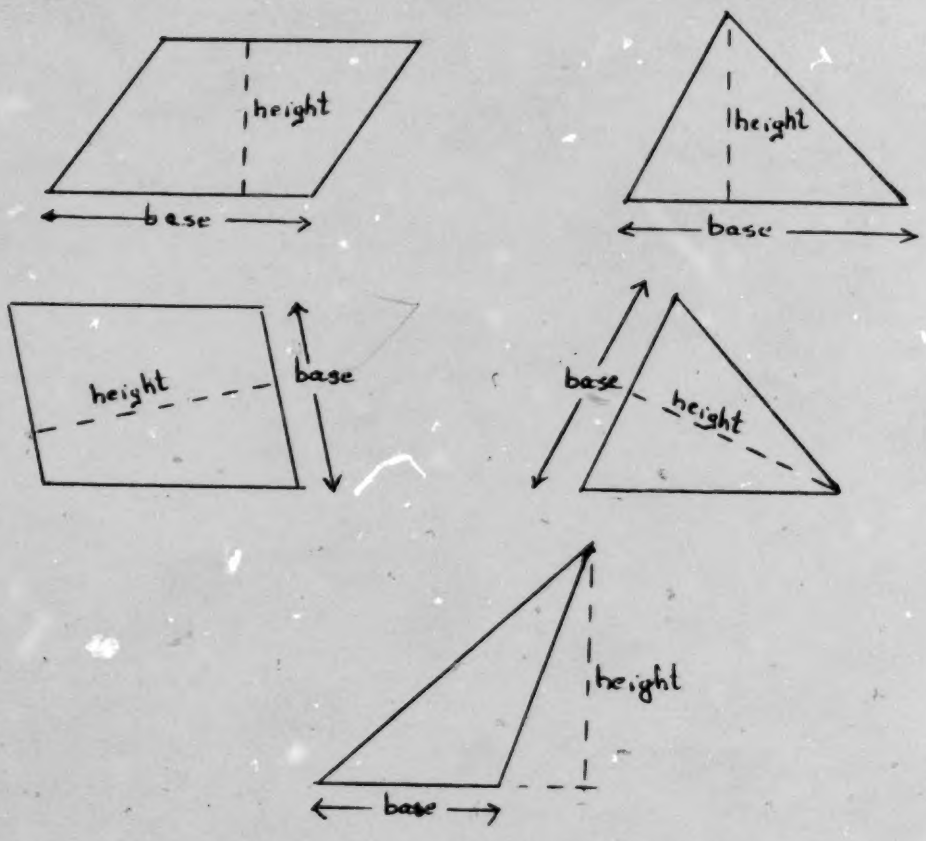
The perimeter of a rectangle with sides of length a and b units is equal to twice the sum of a and b, i.e. $2(a+b)$ or $2a + 2b$. The area of a rectangle with sides a and b units is equal to the product of a and b, i.e. $a \times b$.

Perhaps you are more comfortable expressing the result as:

The area of a rectangle is equal to the product of its base and height.



Throughout the tasks in this activity when reference is made to the base and height of a figure, it is understood that the height is drawn to the base (or base extended) under consideration. For example:



Some of the tasks in this section require use of geoboards. It may be necessary to place several geoboards side by side in order to complete the problem. On the geoboard the unit of length will be either the horizontal or vertical distance between 2 nails. The unit of area to be used when measuring a plane region is the area of a region formed by a square one unit on a side.

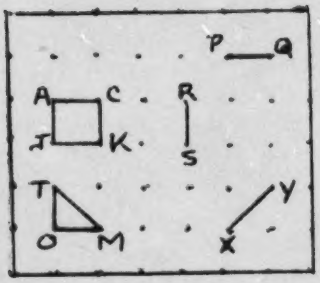


Illustration:

Both PQ and RS measure one unit of length. Note, that the measure of XY is more than one unit long because it is the diagonal distance between 2 nails.

In working with area units, note, in above diagram:

JACK has a measure of 1 area unit while TOM has a measure of ½ area unit.

NOW TURN TO NEXT PAGE IF YOU CHOSE ACTIVITY 1 OR TO PAGE 151 IF YOU CHOSE ACTIVITY 2.

Activity 1

1. Do Tasks 6.1, 6.2, 6.3, and 6.4.
When you have completed each task, check the appropriate box.

TASK	COMPLETED
6.1	
6.2	
6.3	
6.4	

2. Do either Task 6.5 or 6.6
When you have completed the task, check the appropriate box.

TASK	COMPLETED
6.5	
6.6	

3. Do two Post-Assessment Tasks from among Tasks I, II, and III.
When you have completed each task, check the appropriate box.

TASK	COMPLETED
I	
II	
III	

Have you completed two of the post-assessment tasks?

4. You will need the following materials to help you with the tasks in this activity:

Geoboards and rubber bands
Unlined paper, graph paper, dot paper
24 square tiles (congruent)
Cuisenaire rods
Straight edge and scissors
Rectangular solids, cylinders and spheres of various sizes
Dried lima beans
Model of area of circle using liquid

TURN THE PAGE AND BEGIN.

Task 6.1 Square

For each of the geoboard constructions below, record in Table 6.1 the length of the side, perimeter, and area:

a. Construct a square 3 units on a side.
b. Construct a square 2 units on a side.
c. Construct a square 4 units on a side.
d. Construct a square 6 units on a side.

Table 6.1

Length of side	Perimeter	Area
3		
2		
4		
6		

e. From the patterns noted in Table 6.1 predict the perimeter and area of a square with side measuring 4.5 units.
perimeter = _____
area = _____
Check by sketching the square on graph paper and counting the number of units.

f. Generalize your discoveries to predict the perimeter and area of a square whose side measures s units:
1. perimeter of square s units on a side = _____
2. area of square s units on a side = _____

g. Construct the following square on a geoboard. Show that its area is two square units.

Approximate the length of the side of this square_____
Justify this answer in light of f 2 above:

_____

_____

_____

h. Cut out of unlined paper a square 4 inches on a side. Form a new square by folding the corners of the square into the center so that the corners meet at one point. Compare the area of the new square with that of the original square.

What is the area of the new square?

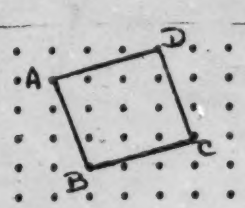
What is the length of a side of the new square?

Repeat this process a second time by folding in the corners of the new square to form a new "new square." Continue this process a third time; a fourth time.
Tabulate your results below.

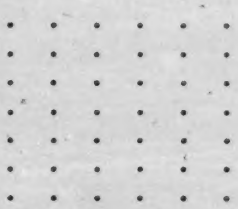
Length of Side	Area	New Area	Length of New Side

What patterns do you notice?

i. Find the area of square ABCD in the adjoining diagram. Describe in words, two alternate approaches for finding the area.

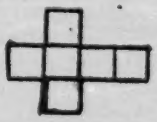


*j. Can you construct a square whose area is 5 square units? If so, record your result below. If not, state the reason why.



k. The adjoining configuration of 6 squares can be folded to form a closed box (a cube). If the side of each square has a length of 2 inches, what is the surface area of the box?

Surface area = _____



1. The faces of a cube are all squares. What is the surface area of a cube whose side measures 3 inches?

_____

Using a square inch as a measuring unit, why is the measure of the surface of the cube not equal to 27?

_____

_____

_____

What is the surface area of a cube whose side measures e inches?

Surface area of cube = _____

TURN TO THE NEXT PAGE AND CHECK YOUR RESPONSES.

Discussion of Responses

Task 6.1

Table 6.1

Length of Side	Perimeter	Area
3	12	9
2	8	4
4	16	16
6	24	36

e. Perimeter = 18 units (units of length)
Area = 20.25 square units (units of area)

f. Perimeter = $4s$
Area = s^2

g. By dividing up the figure with rubber bands, we obtain four congruent triangles. The area of each triangular region is ½ square unit. Hence the area of the square region is 4 (½) or 2 square units.

Length of side is approximately 1.4 units

(NOTE: Length of side could be expressed as $\sqrt{2}$
You may recall having learned a mathematical principle called the Pythagorean Theorem which states that in a right triangle the sum of the squares of the legs is equal to the square of the hypotenuse, i.e. $(\text{leg } 1)^2 + (\text{leg } 2)^2 = (\text{hypotenuse})^2$

So in this instance, in one of the right triangles, each leg is 1 unit, hence

$(\text{hypotenuse})^2 = 1^2 + 1^2$

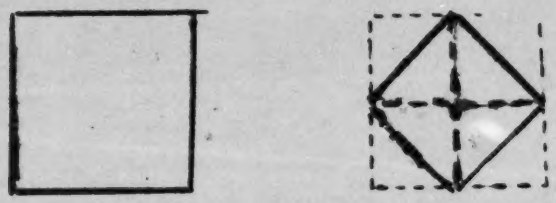
$(\text{hypotenuse})^2 = 1 + 1$

$(\text{hypotenuse})^2 = 2$

Thus hypotenuse $= \sqrt{2}$

Justification: $s^2 = (1.4)^2 = 1.96$
or approximately 2 square units

(NOTE: $s^2 = (\sqrt{2})^2 = 2$)



h. Area of new square is ½ area of original square.

Length of side of new square is approximately 2.8 inches.

(NOTE: Length of side could be expressed as $2\sqrt{2}$)

Length of Side (S)	Area	New Area	Length of New Side (S)
4	16	8	2.8 or $2\sqrt{2}$
2.8	8	4	2
2	4	2	1.4 or $\sqrt{2}$
1.4	2	1	1

Patterns:

To find the length of the side of the new square formed by halving the original square, divide the length of the original square by 1.4 (or by $\sqrt{2}$)

or

Original Area = A or S^2

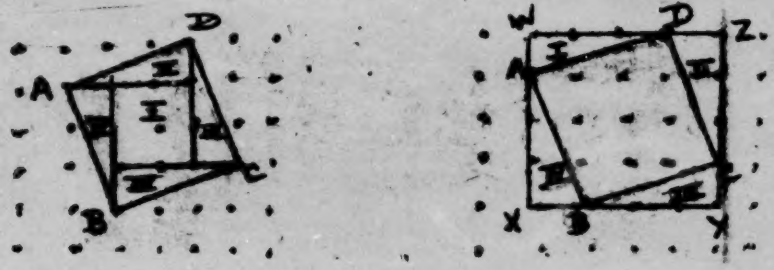
New Area = $\frac{1}{2}A$ or $\frac{1}{2}S^2$

Length of new side : n
So new area also equals n^2

$$\therefore n^2 = \frac{S^2}{2}$$

$$n = \frac{S}{\sqrt{2}}$$

i. Area = 10 square units



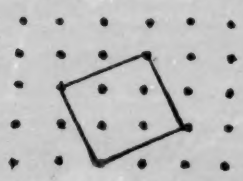
Method 1: Area of region I is 4 square units
Area of each of regions II through V is 1½ square units

Total area of ABCD = 4 + 4 (1½)
= 4 + 6 = 10 square units.

Method 2: Area of region WXYZ = 16 square units
Area of each of regions I through IV is 1½ square units

Total area of ABCD = 16 - 4 (1½)
= 16 - 6 = 10 square units

j.



k. Surface area = 24 square inches

l. Surface area = 54 square inches

Area of each face is 9 square inches and there are 6 faces
Hence 6 x 9 = 54 square inches

If you obtained 27, you multiplied 3 by 3 by 3 which would represent 27 cubic inches or the VOLUME of the cube (not the surface area)

Surface area of cube = $6\ e^2$

Have you completed Task 6.1 satisfactorily?

Yes - Proceed to Task 6.2

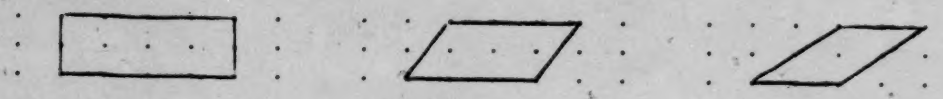
No - Is this your first try?

Yes - Read one of the suggested texts on page 151 and redo exercise.

No - Do Task 6.2 or see instructor.

Task 6.2 Parallelogram

a. On a geoboard construct a rectangle and then change it to a parallelogram as indicated in the adjoining diagram.



How does the area of each of the parallelograms compare with that of the rectangle?

_____

b. For each of the geoboard constructions below, record in Table 6.3 the length of the base, the height and the area enclosed by the figure.

1. Construct a parallelogram (which is not a rectangle) enclosing an area of 3 square units. Is there more than one possibility?

_____

2. Construct a parallelogram (which is not a rectangle) having an area of 8 square units. Is there more than one possibility?

_____

3. Construct a rectangle and a parallelogram (which is not a rectangle) on the same base each having an area of 16 square units.

4. Construct a rectangle and a parallelogram (which is not a rectangle) each having a base of 5 units and an area of 20 square units.

Table 6.2

Length of base	Height	Area
		3
		8
		16
5		20

c. From the patterns noted above, predict the area of a parallelogram (which is not a rectangle) with base length 6 units and height of 3.5 units.

Area = _____

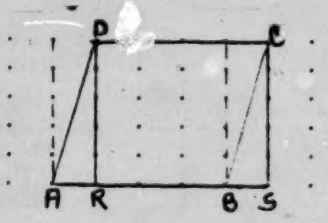
Predict the number of units in the width of a rectangle having the same area as this parallelogram if its length is 10 units.

Width = _____

d. Generalize your discoveries to predict the area of a parallelogram (which is not a rectangle) with base of length a units and height of b units.

Area of parallelogram = _____

e. In the adjoining figure justify (without counting units) why parallelogram ABCD has the same area as rectangle RSCD.



_____

_____

_____

TURN TO NEXT PAGE AND CHECK YOUR RESPONSES.

Discussion of Responses

Task 6.2

a. Area of all 3 figures are equivalent.

b. Table 6.2

Length of base	Height	Area
3	1	3
8	1	8
4	2	8
2	4	8
1	8	8
16	1	16
8	2	16
4	4	16
2	8	16
1	16	16
5 -	4	20

1. Many possibilities.

2. Many possibilities.

c. Area = 21 square units
Width = 2.1 units

d. Area of parallelogram = a x b

e. Parallelogram ABCD is made up of triangle ARD and quadrilateral RBCD.
Rectangle RSCD is made up of triangle BSC and quadrilateral RBCD.
Triangles ARD and BSC are congruent and thus have equivalent areas.
Hence parallelogram ABCD and rectangle RSCD have equivalent areas.

Have you completed Task 6.2 satisfactorily?

Yes - Proceed to Task 6.3

No - Is this your first try?

Yes - Read one of the suggested texts on page 151 and redo exercise.

No - See instructor.

Task 6.3 Triangle

a. If one diagonal is drawn in each of the parallelograms and rectangles in Activity 3 a-b, what is true of the two triangles formed in each case?

_____

In each case, how does the area of one of these triangles compare with the area of the whole figure?

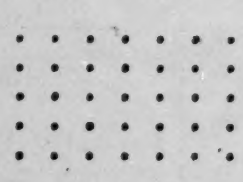
_____

b. For each of the geoboard activities below, record in Table 6.4 the length of the base, the height and the area of the triangle.

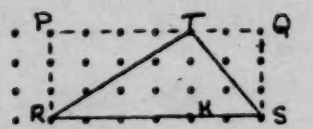
1. Construct a right triangle with legs of length 2 units and 3 units.

2. Construct a triangle having an area of 1 square unit. Can you construct an obtuse triangle with an area of one square unit?

_____Illustrate.



3. Construct triangle RST as in the adjoining figure.



If TK is drawn, how does the area of triangle RKT compare with the area of rectangle PRKT?

_____

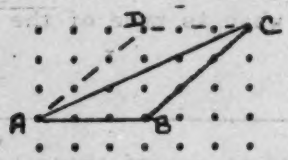
How does the area of triangle SKT compare with the area of rectangle QSKT?

_____

How does the area of triangle RST compare with the area of rectangle PRSQ?

_____

4. Construct triangle ABC as in the adjoining figure.



How does the area of triangle ABC compare with the area of parallelogram ABCD?

Table 6.4

Length of base	Height	Area
3	2	
		1
6		
3		

c. From the patterns noted above, predict the area of a triangle with height of 2.5 units and base length 4 units.

Area = _____

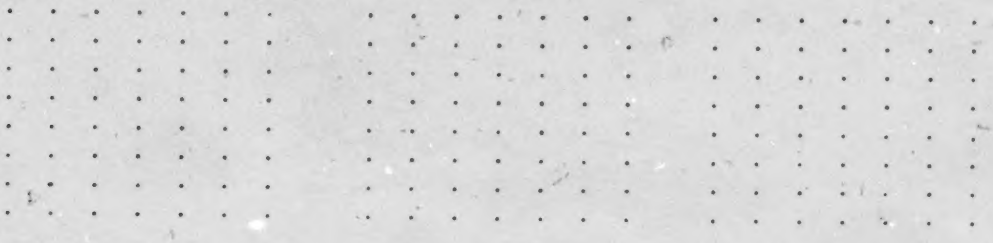
d. Generalizing from your discoveries, predict the area of a triangle with a base length a and height of b units.

Area of triangle = _____

e. Construct a triangle and a square which have equal area. Record your result below.

Which of the two figures has the smaller perimeter?

*f. Explore the possibilities of constructing a right triangle, an isosceles triangle, and an equilateral triangle on a geoboard so that each triangle has an area of 6 square units. Record your results below.



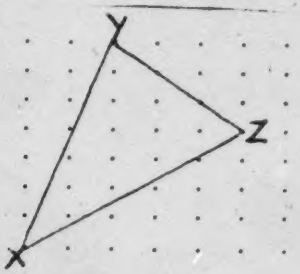
What conclusions have you reached?

_____

_____

_____

g. Find the area of triangle XYZ in the adjoining diagram.



Area = _____

TURN TO NEXT PAGE AND CHECK YOUR RESPONSES.

Discussion of Responses

Task 6.3

Possible Answers:

a. Triangles are congruent.
Triangles are equal in area.
Area of one of triangles is ½ of the area of the whole figure.

b.

Table 6.3

Length of base	Height	Area
3	2	3
2 1	1 2	1 1
6	3	9
3	3	4.5

2.

Obtuse triangle with an area of one square unit.

3. Area of Δ RKT is ½ area of rectangle PRKT

   Area of Δ SKT is ½ area of rectangle QSKT

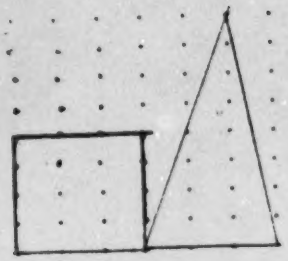
   Area of Δ RST is ½ area of rectangle PRSQ

4. Area of Δ ABC is ½ area of parallelogram ABCD

c. Area = 5 square units

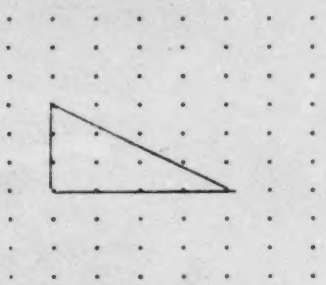
d. Area of triangle = ½ a x b

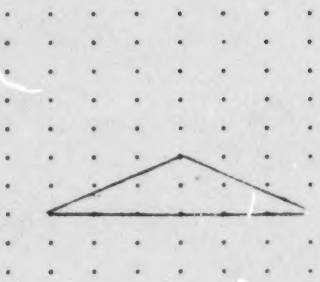
e. One possible construction is:



Square has the smaller perimeter

*f. Some possible constructions are:



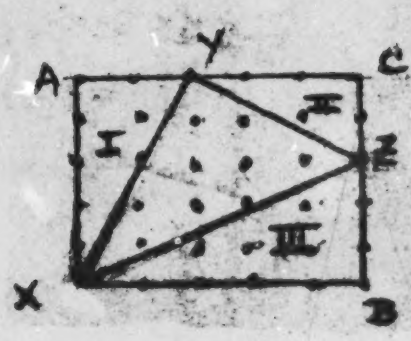


Conclusions:

It is possible to construct several right triangles and several isosceles triangles having an area of 6 square units.

It is impossible to construct on the rectangular geoboard an equilateral triangle having an area of 6 square units.

g.



Build rectangle AXBC around triangle XYZ
Area of rectangle AXBC = 25 square units

Area of triangle I = 5 square units.
Area of triangle II = 3 square units.
Area of triangle III = 7.5 square units.

Total area of triangles I, II, and III = 15. 5 square units.

Hence, area of triangle XYZ = 25 - 15.5
area of triangle XYZ = 9.5 square units.

Have you completed Task 6.3 satisfactorily?

Yes - Proceed to Task 6.4

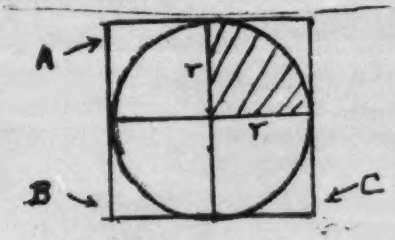
No - Is this your first try?

Yes - Read one of the suggested texts on page 151 and redo exercise.

No - See instructor.

Task 6.4 Circle

a. On a sheet of graph paper, draw a circle of radius 7 units. Then draw a square around the circle and two perpendicular diameters as indicated in the adjoining diagram.

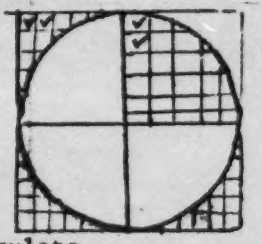


r = length of the radius of circle

We wish to compare the area of the circle to the area of the four small squares.
What is the area of each of the small squares if the length of the side of the square is r units?

_____

Clearly the area of the circle is less than the sum of the areas of the four squares. i.e. Area of circle $< 4r^2$. But how much less? In order to investigate this, we are going to try and squeeze one fourth (shaded portion) of the circular region into three unused corners (regions A, B, C,) of the large square. To do this, check off units (or partial units) in the shaded portion of the circular region with corresponding units (or partial units) in the three corner regions. Continue this until you have checked off all the units or partial units in the three corner regions. Have you any unchecked units left in the shaded portion of the circular region?



_____

Repeat this experiment for a circle whose radius measures 4 units.

b. From these experiments, what hypothesis could you now formulate concerning the area of a circle of radius r units in relation to the areas of squares with sides of length r units?

_____

_____

_____

_____

c. Draw a circular region of radius 14 units on graph paper. Cover the circular region with a single layer of lima beans. Using the same set of lima beans, see how many squares having side length of 14 units you can cover with a single layer of the beans. On the basis of this experiment, predict the number of squares having side length of 10 centimeters that you can cover using a set of beans which cover a circular region with a radius of 10 centimeters.

_____

Compare your result to that obtained by using the mathematics laboratory model of the area of a circle which contains liquid.

d. Cut out two paper circular regions with 7 inch diameter. Cut up one region into 8 equivalent pieces. Rearrange the pieces into a new configuration resembling a "parallelogram."

What is the length of the base of the "parallelogram"?

_____

(Compare this length to the circumference of the circle.)

What is the height of the "parallelogram"?

_____

What is the area of the "parallelogram"?

_____

Hence what is the area of the circle?

_____

Using this same procedure, what would be the area of a circle if the length of its radius is r units?
Area of circle = _____

Comment on this result in relation to your hypothesis stated in part b above:

_____

_____

TURN TO NEXT PAGE AND CHECK YOUR RESPONSES.

Discussion of Responses

Task 6.4

a. Area of each small square = r^2

Yes

b. The area of a circle of radius r units is a little more than the areas of three squares with sides of length r units.

c. A little more than 3 squares.

d. Length of base of "parallelogram" is equivalent to ½ length of circumference of circle (i.e. ½ · 7π) or length of base of "parallelogram" = 3.5π.

(For explanation of finding the circumference of a circle, see Sadlier, Contemporary Mathematics 7, page 374 - examples 6-10.)

Height of "parallelogram" = 3.5
Area of "parallelogram" = base x height
Area of "parallelogram" = 3.5π x 3.5
Area of "parallelogram" = 12.25π

Hence, area of circle = 12.25π

Area of circle with radius r units = πr^2

Comment: The number π has a value of a "little more than 3" (i.e. π = 3.1416...)
r^2 represents the area of a square with length of side of r units. Hence, the expression πr^2 represents a quantity which is a little more than the areas of three squares with sides of length r units.

Have you completed Task 6.4 satisfactorily?

Yes - Proceed to either Task 6.5, page 144, or Task 6.6, page 147.

No - Is this your first try?

Yes - Read one of the suggested texts on page 151 redo exercise.

No - See instructor.

Task 6.5 Right Circular Cylinder

Take two cylindrical fruit, soup or vegetable cans of the same size. Take off the labels by cutting one label vertically and the other label obliquely as indicated in the diagram.

vertical cut

oblique cut

Flatten out the labels and note their shapes.

Find the area of each of the labels. _____ _____

What is the length of the labels in relation to the original cylinders?

_____

What is the height of the labels in relation to the original cylinders?

_____

How can you find the area of each label without removing the labels?

_____

_____

_____

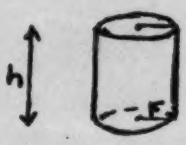
Find the area of the two end faces of <u>one</u> can.

_____

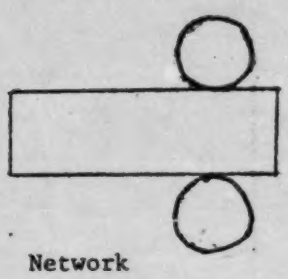
What is the total surface area of <u>one</u> of the cans?

_____

The network of a right circular cylinder of radius r and height h is shown in the adjoining diagram.



Cylinder



Network

Considering this network, in terms of r and h:

What is the length of the rectangle? _____

What is the width of the rectangle? _____

What is the area of the rectangle? _____

What is the area of the two circles? _____

What is the total surface area of the cylinder (i.e. the area of the network)?

_____

_____

TURN TO THE NEXT PAGE AND CHECK YOUR RESPONSES.

Discussion of Responses

Task 6.5

The shape of one label is a rectangle, the other is a parallelogram. Your two labels should have the same area. (See also Element V, Post-Assessment Task I, part 2, page 115 .)

Length of label is the same as the circumference of the base of the cylinder.

Height of label is the same as the height of the cylinder.

To find the area of a label without removing it from can, find the product of the length of the circumference of base of can and the height of the can.

Area of two end faces of one can will be $2 \times \pi \times \text{radius}^2$

Total surface area of one can is the sum of the area of your label and the areas of the two end faces of your can.

Length of rectangle = $2\pi r$

Width of rectangle = h

Area of rectangle = $2\pi rh$

Area of two circles = $2\pi r^2$

Total surface area of cylinder = $2\pi rh + 2\pi r^2$

Have you completed Task 6.5 satisfactorily?

Yes - Return to page 123 and follow remaining directions.

No - Is this your first try?

Yes - Read one of the suggested texts on page 151 and redo exercise.

No - Do Task 6.6 or see instructor.

Task 6.6 Sphere

Your task is to collect data which will assist you in discovering methods for finding the surface area of a sphere. Three approaches are suggested. In each instance use spheres (balls, oranges,...) of varying sizes and record the length of the radius and the area.

a. Carefully peel an orange and lay the peel on graph paper. See how many units of graph paper the peel covers. Repeat with oranges of several sizes and record results in Table 6.6a.

Table 6.6a

Length of radius (in terms of graph paper units)	Area

b. Measure the diameter of a sphere. Using the length of the radius of this sphere, construct on unlined paper 5 or 6 great circles for this sphere. Cut out these circular regions. How many of these paper circular regions are needed to cover the entire surface of the sphere?

_____

Actually cut up the circular regions and paste the pieces on the sphere to see how many are really needed to cover the sphere completely. Repeat this process for several spheres of different sizes; record results in Table 6.6b.

Table 6.6b

Length of radius	No. of great circular regions needed to cover this sphere

If the radius of a sphere is r, what is the area of one great circle of this sphere?

_____

From the last experiment which used great circles of a sphere, formulate a hypothesis for finding the surface area of a sphere of radius r units.

Hypothesis: _____

_____

Check this hypothesis to see if it holds true for the data you collected in parts a or b above.

TURN TO THE NEXT PAGE AND CHECK YOUR RESPONSES.

Discussion of Responses

Task 6.6

a. Some possible entries:

Table 6.6a

Length of radius	Approximate area in square units
5	314
6	452
8	804
12	1808

b. Four

Some possible entries:

Table 6.6b

Length of radius	No. of great circular regions needed to cover the sphere
4	4
6	4
7	4
10	4
12	4
15	4

c. (continued)

Area of one great circle = πr^2

Possible hypothesis:

The surface area of a sphere of radius r units is equivalent to areas of four great circles of radius r.

or

Surface area of sphere = $4\pi r^2$
(of radius r units)

Have you completed Task 6.6 satisfactorily?

Yes - Return to page 123 and follow remaining directions.

No - Is this your first try?

Yes - Read one of the suggested texts on page 151 and redo exercise.

No - If you have not done Task 6.6, do it or see instructor.

Activity 2

Select three books from the list below and read the indicated pages.

1. Biggs, Freedom to Learn, pp. 9-13.

2. Bouwsma, Geometry for Teachers, Chapter 8.

3. Hooten, Mahaffey, Elementary Mathematics Laboratory Experiences, pp. 24-43, pp. 73-82.

4. Keedy, Nelson, Geometry, A Modern Introduction, Second Edition. pp. 152-166.

5. Ringenberg, Informal Geometry, Chapter 4.

6. Sadlier, Contemporary Mathematics 7, Teacher's Edition, pp. 376-382, pp. 401-411.

7. Sawyer, Visions in Elementary Mathematics, Chapter 6.

8. SMSG, Mathematics for the Elementary School, Grade 5, Part II, Chapter 6.

9. SMG, Volume IX, Chapters 27, 28.

10. SMP, Book B pp. 41-49
    Book C pp. 7-23
    Book E pp. 124-125, 144
    Book F pp. 85-90.

11. UICSM, Motion Geometry, Book IV, pp. 1-55, pp. 113-116, pp. 121-122.

TURN TO THE POST-ASSESSMENT TASKS (BEGINNING ON PAGE 152) AND CHOOSE TWO.

Post-Assessment Task I Pick's Theorem

a. Discover Pick's theorem by doing either of the following exercises:

1. SMP, Book C, pp. 21-23.

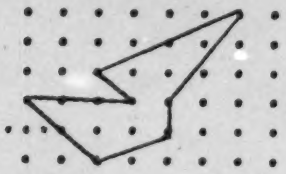
2. Hooten, Mahaffey, Elementary Mathematics Laboratory Experiences, pp. 41-43.

b. Using Pick's theorem, verify the answer found for the area of:

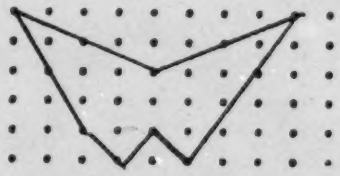
1. square ABCD in Activity 6.1 i;

2. triangle XYZ in Activity 6.4g.

c. Using Pick's theorem find the area of each of the following regions:



Area = _____



Area = _____

d. Verify your results in part c by finding the area of each of the figures by an alternate method. Explain alternate method you used:

_____

_____

_____

_____

_____

_____

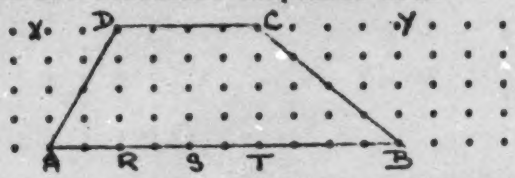
_____

_____

GO ON TO POST-ASSESSMENT TASK II or III.

Post-Assessment Task II Trapezoid

a. On a geoboard construct trapezoid ABCD



1. How does the area of trapezoid ABCD compare with the area of rectangle ABYX?

_____

of rectangle RTCD?

_____

What is the area of rectangle ABYX?

_____

What is the area of rectangle RTCD?

_____

Hence the area of the trapezoid ABCD is between what two bounds (limits)?

_____

2. How does the area of trapezoid ABCD compare with the sum of the areas of parallelogram ASCD and triangle SBC?

_____

What is the area of parallelogram ASCD?

_____

What is the area of triangle SBC?

_____

3. How does the area of trapezoid ABCD compare with the sum of the areas of rectangle RTCD and triangles ARD and BTC?

_____

What is the area of rectangle RTCD?

_____

What is the area of triangle ARD?

_____

What is the area of triangle BTC?

_____

4. How does the area of trapezoid ABCD compare with the sum of the areas of triangles ABD and BCD?

_____

What is the area of triangle ABD?

_____

What is the area of triangle BCD?

_____

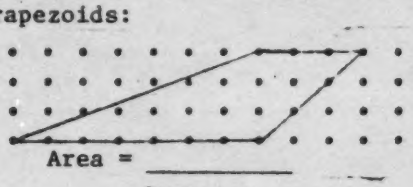
5. From the above what is the area of trapezoid ABCD?

_____

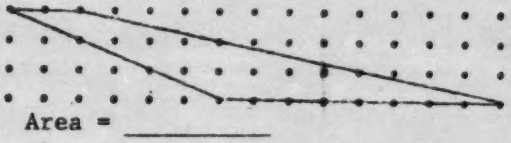
Is your answer consistent with (1) - (4) above?

_____

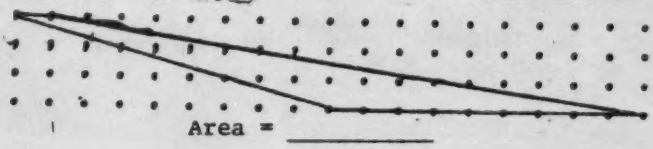
b. By dividing up the region into rectangles, parallelograms or triangles or by surrounding the region with rectangles or parallelograms, find the area of each of the following trapezoids:



Area = _____



Area = _____

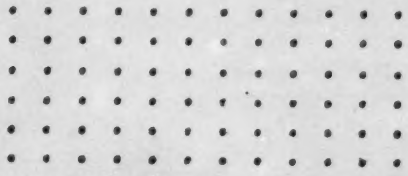


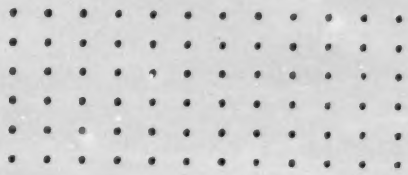
Area = _____

c. On a geoboard construct a trapezoid with an area of 5 square units. Record your result below. Is there more than one possibility?

_____

If so, record another.





d. Construct at least 5 or 6 different trapezoids on your geoboard. Find the area of each using one of the methods discussed in part a above. Record in Table II the following data for each trapezoid: length of one base, length of other base, average length of two bases, height, area.

Table II

Length of One Base	Length of Other Base	Average Length of Two Bases	Height	Area

From your data formulate a hypothesis for finding the area of a trapezoid in terms of the average length of the bases and the height.

Hypothesis:

_____

_____

Test your hypothesis to see if by using it you obtain the same answers for the problems you previously solved by other methods in parts (a), (b), and (c) above.

e. Given a trapezoid where the average length of the two bases is m and the height is h express the area of the trapezoid in terms of m and h.

Area = _____

f. Does the relationship for finding area that you stated in part (e) hold true if applied to finding the area of a parallelogram?

_____ a rectangle?_____ a triangle?_____

Justify your answers:

_____

_____

_____

_____

_____

_____

_____

Have you completed TWO Post-Assessment Tasks for Activity 6?

Yes - Turn to page 160 and follow directions.

No - Proceed to Post-Assessment Task III or I.

Post-Assessment Task III Equivalent Areas

a. On a geoboard construct a rectangle, a parallelogram (which is not a rectangle), an obtuse triangle, an isosceles triangle, a trapezoid and a hexagon so that they all have equivalent areas. Record your results on dot paper. (next page)

b. Construct on a geoboard 6 different triangles - scalene, isosceles, right, obtuse, ...In each triangle construct a median.

What is true of the two triangles formed in each case?

_____

Record your results on dot paper. (pages 158-159)

Formulate a hypothesis that this experiment suggests.

Hypothesis:

_____

_____

_____

_____

How might you justify this hypothesis?

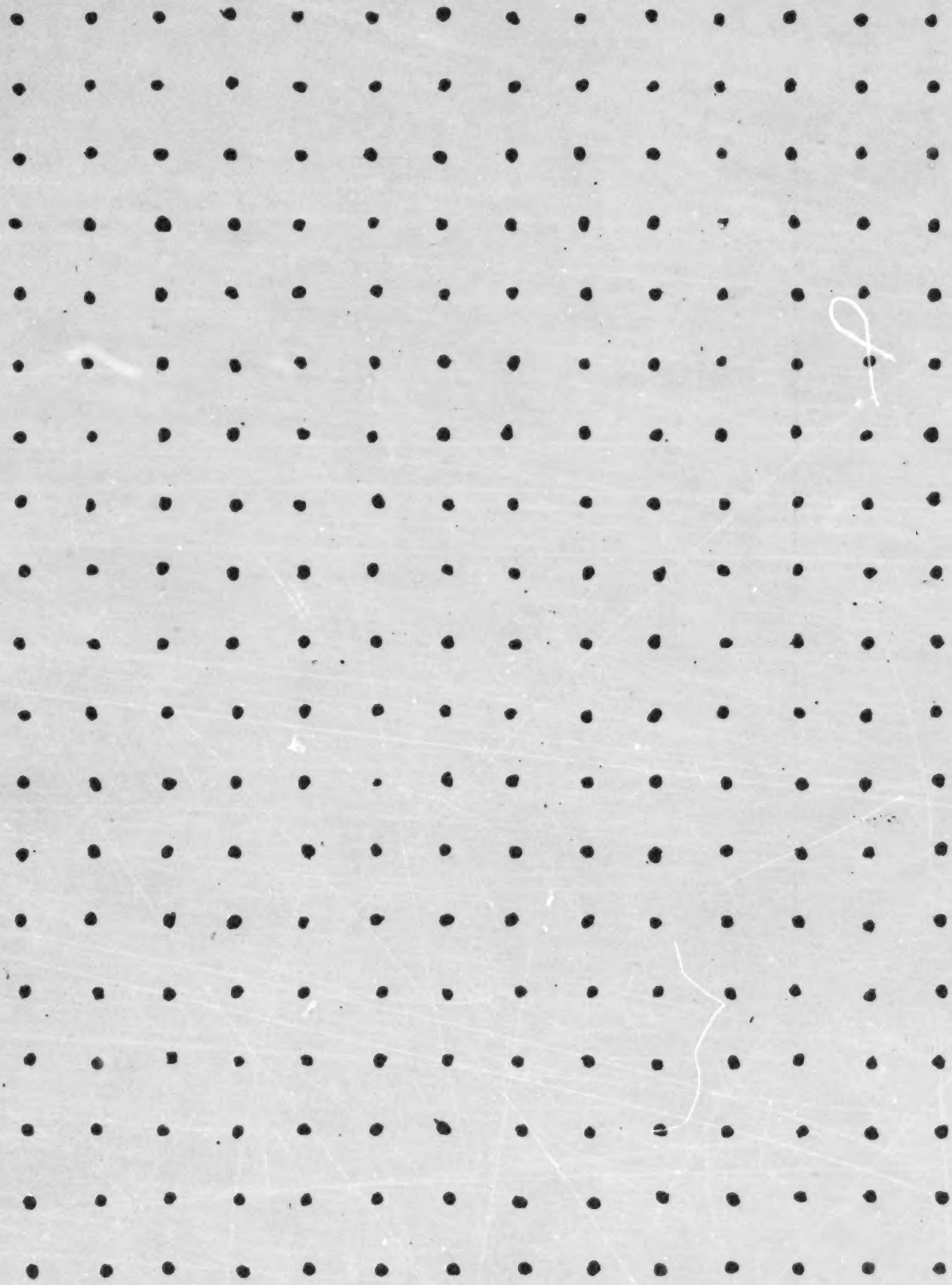
_____

_____

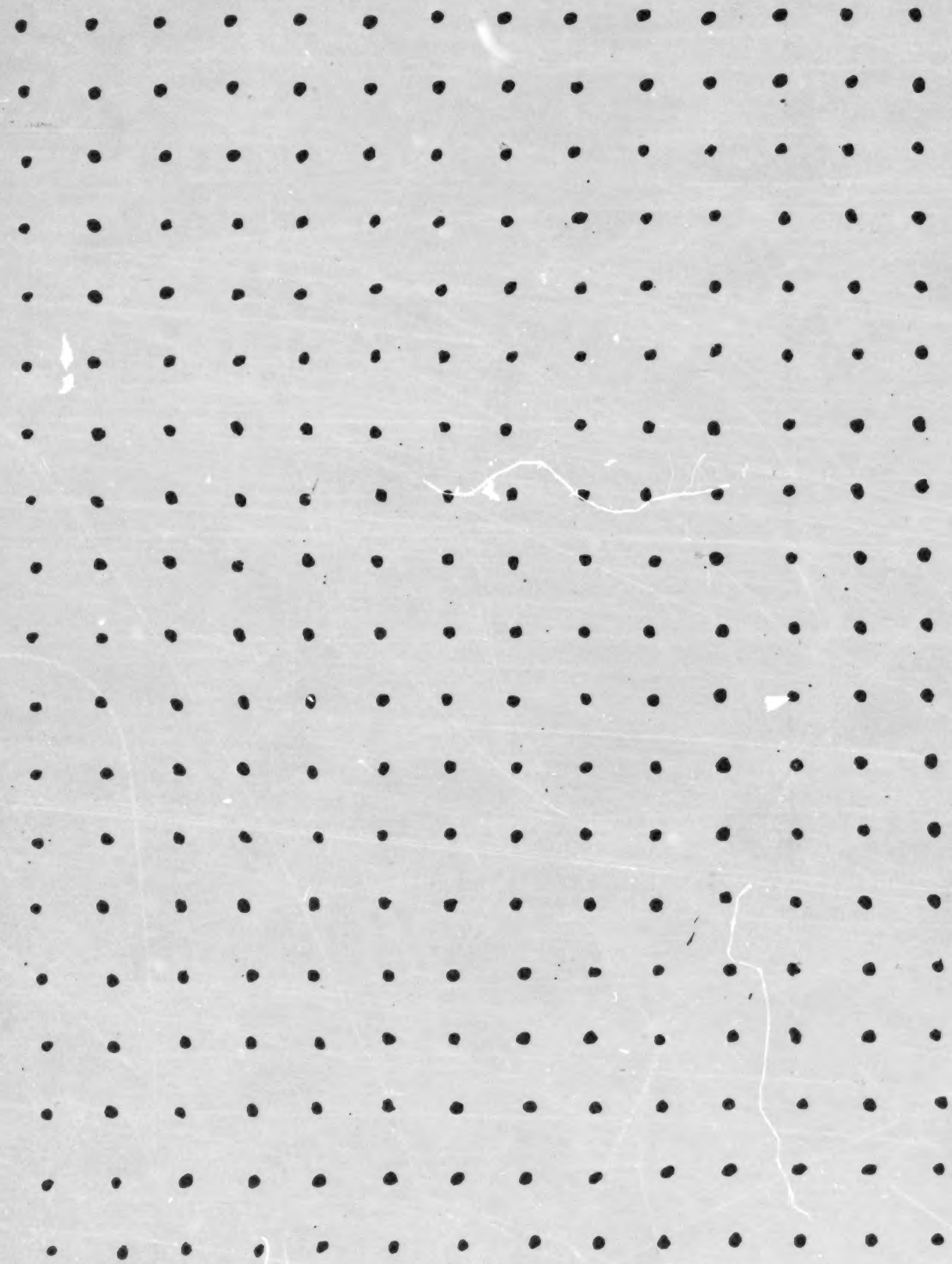
_____

NOW TURN TO DIRECTIONS ON PAGE 160.

Record results for IIIa.



Record results for IIIb.



Directions

1. Check your responses to <u>two</u> post-assessment tasks on pages 12-16. in the Response Manual.

2. Your responses to

   a. Task I agree? Yes_____ No _____

   b. Task II agree? Yes_____ No _____

   c. Task III agree? Yes_____ No _____

3. If you have checked NO to any part of the above questions, read one of the suggested readings on page 151, and redo the Post-Assessment Task. Then go on to Direction 4.

4. If you have checked YES to two tasks in question 2, go back to page 10.

5. On your Decision Chart, place a check in column 3 to indicate that you have completed the original activity.

6. Now turn to Section II and complete Activity C on pages 195-200.

ELEMENT VII

Objective: Given problem situations, you will be able both to identify those involving the concept of area and to produce solutions to those problems.

This element is designed to help you to identify problems involving the concept of area and to learn how to solve those problems. You will meet the objective of this element by completing one of two tasks. You may choose Activity 1, the specially written material, or Activity 2, outside reading. When you have completed either one, take the post-assessment, which consists of three tasks. If you achieve a satisfactory score on these, you can consider that you have successfully met the goal of this element. Before you choose your activity, read the following background material.

Background

Tiling or carpeting a floor, fencing in a region, covering a wall, making a lampshade--all these are problems which involve the concept of area.

Here are two examples for you to try:

Example 1: A painter uses an 8-inch-wide paintbrush. With one stroke of his brush, he usually covers a vertical distance of about 3 feet.

a. How many such strokes would be needed to cover a wall which measures 9 feet by 12 feet?

b. Does the painter cover the same amount of surface if his stroke is less steady and resembles the shape at the right while still covering a vertical distance of 3 feet?

Example 2: The floor of a room is 18 feet long and 12 feet wide.

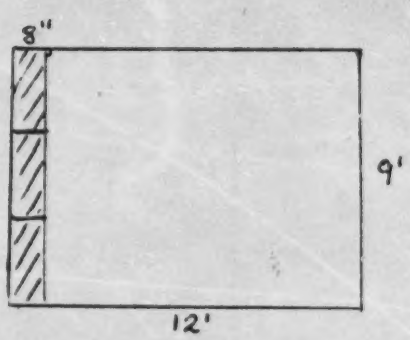
a. How many 12 inch square tiles would be needed to tile the floor of this room?

b. How many 9 inch square tiles would be needed to tile the floor of this room?

TURN TO THE NEXT PAGE FOR THE ANSWERS.

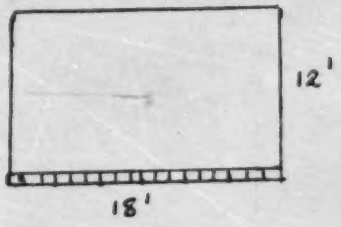
In example 1 a, it is clear that the painter would need 3 strokes to do one vertical distance of 9 feet (see diagram). Since each stroke is only 8 inches wide it is necessary to find how many 8 inch (or 2/3 foot) units there are in 12 feet -

$$12 \div \frac{2}{3} = 12 \times \frac{3}{2} = 18$$



Thus the painter would need 18 x 3 or 54 strokes to cover the wall. In 1 b do you agree that the painter covers the same amount of surface with his unsteady stroke as with his vertical stroke? If you do not agree, reread the discussion in Element II, Task 2.2, and the discussion on equivalence of area in Element V, Task 5.3.

In example 2 a, do you agree there are 216 one foot (or 12 inch) square tiles needed? As shown in the diagram, 18 one foot square tiles would be needed in one horizontal row and 12 such rows are needed. Thus, 18 x 12 = 216 tiles. Similarly in example 2 b, 24 nine inch (or 3/4 foot) square tiles would be needed in one horizontal row and 16 such rows (i.e. $12 \div \frac{3}{4} = 16$)

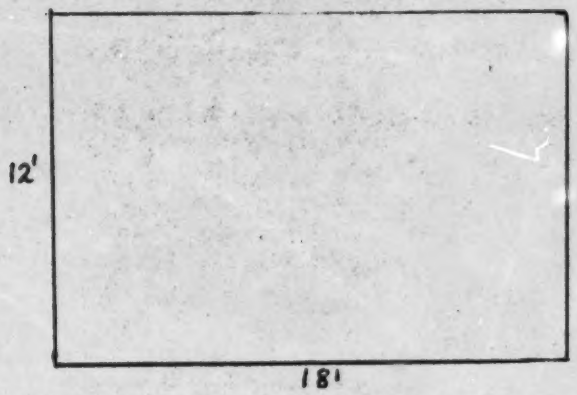


are needed. Thus since 24 x 16 = 288, the number of nine inch tiles required is 288.

IF YOU HAVE CHOSEN ACTIVITY 1, START IT BELOW. IF YOU HAVE CHOSEN ACTIVITY 2, TURN TO PAGE 171.

Activity 1. Do all the tasks. Then check your responses to all of them at once, on pages 169-170.

Task 7.1. The floor of a room is 18 feet long and 12 feet wide.



a. How many 6-inch-square tiles would be needed to tile the floor?

Answer_____

b. How many oblong tiles 12 inches by 6 inches would be needed to tile the floor of the same room?

Answer_____

Task 7.2. Answer either 1 or 2.

1. Room A is 16 feet long, 14 feet wide and 8.5 feet high.

   Room B is 20 feet long, 12 feet wide and 8 feet high.

   a. Which room has the most wall space?

   _____

   Justify your answer.

   _____

   _____

   _____

   b. Which room has the most floor space?

   _____

Justify your answer.

_____

_____

_____

2. A room 18 feet long, 10 feet wide and 8 feet high is to be redecorated. The walls and ceiling are to be painted the same color (one coat of paint). The floor is to be tiled with 12 inch square tiles.

   a. How many quarts of paint will be needed if one quart covers approximately 225 square feet?

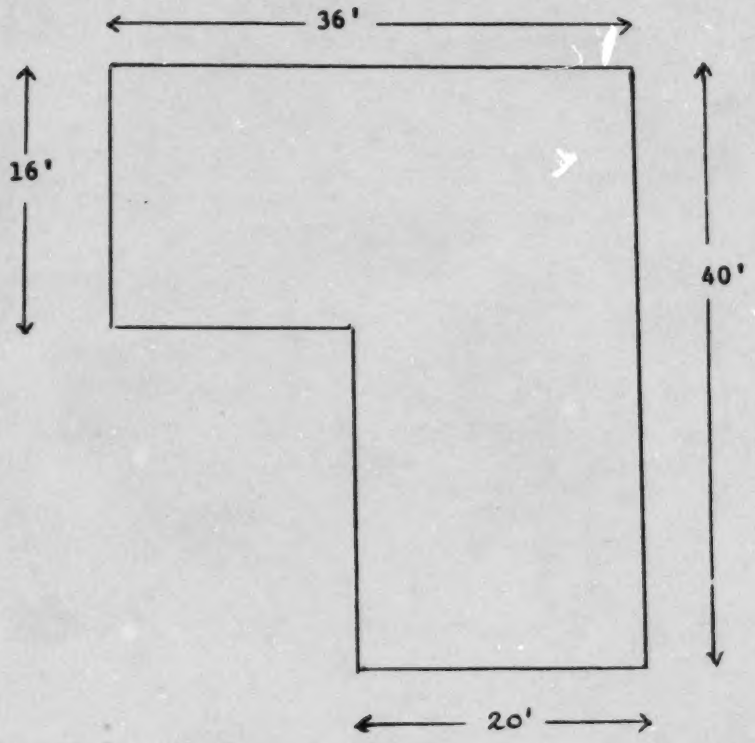
Answer_____

If the tiles cost 19 cents each, how much will it cost to tile the floor?

Answer_____

Task 7.3. Given the following floor plan of a classroom, answer the question.

How many sheets of 4 foot by 8 foot plywood are needed to cover the floor?



Task 7.4. If you had 36 feet of fencing and wished to make a rectangular pen for a puppy, what dimensions should the rectangular shape have in order to allow the puppy the greatest area to run in?

Answer_____

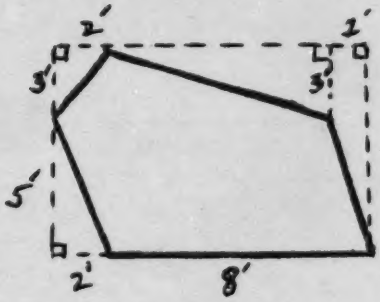
Are there any other shapes that would give even more area?

_____

_____

_____

Task 7.5. Suppose that you wanted to buy just enough carpet for the bold-line figure below. Using the given marked lengths, find the area of this figure.



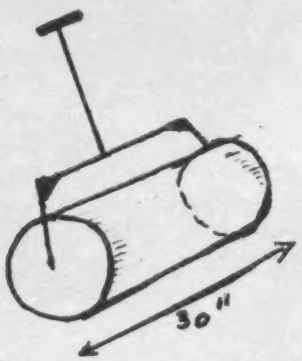
Answer_____

Task 7.6. What is the length of the radius of a circle if the number of inches in its perimeter is equal to the number of square inches in its area?

Answer_____

Task 7.7. A lawn roller (used to roll lawns) is in the form of a right circular cylinder having a radius of 14" and a length of 30".

How many square feet of lawn are rolled in one revolution of the roller?



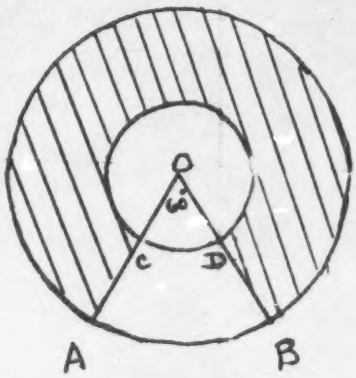
Answer_____

Task 7.8. The flat pattern in the accompanying diagram is used to make a lampshade. The two circles have the same center and radii of 12" and 4".
The measure of angle AOB is 60 degrees.
The minor sector and the remainder of the interior of the small circle are removed and discarded.
Then to form the lampshade, AC is fastened to BD without waste.

Trace the pattern and cut it out. Form a paper model of the lampshade.

Find the area of the outer surface of the lampshade. (i.e. find the area of the shaded portion).

Find the circumference perimeter of the lower base (the larger base) of the lampshade.



Discussion of Responses

Task 7.1

a. 864

b. 432

Task 7.2

1. a. Room B has the most wall space.

Wall space of Room A = 510 square feet.
Wall space of Room B = 512 square feet.

b. Room B has the most floor space.

Floor space of Room A = 224 square feet.
Floor space of Room B = 240 square feet.

2. a. Approximately 3 quarts.

b. $32.20

Task 7.3

33 sheets are needed.

Task 7.4

A square 9 feet on a side; area is 81 square feet.

A circle with circumference of 36 feet would have an area of approximately 103 square feet.

Task 7.5

52 square feet

Task 7.6

Length of radius = 2 inches
(r = o would also satisfy the conditions of problem but is a trivial case)

Task 7.7

Approximately 2640 square inches or 18.3 square feet.

Discussion of Responses (continued)

Task 7.8

Area of outer surface of lampshade is approximately 335 square inches.

Circumference of larger base is 20π or approximately 63 inches.

Have you completed the tasks in Activity 7 satisfactorily?

Yes - Proceed to Post-Assessment for Element VII, page 172.

No - Is this your first try?

Yes - Read one of the suggested texts on page 171 and redo exercise.

No - See instructor.

Activity 2

Read the material suggested below, then turn to the post-assessment for Element VII on page 172.

1. Johnson, et al, Applications in Mathematics, See Index under "Area".

2. Nuffield, Problems, red set, problems 17-20, 22.

3. Nuffield, Problems, green set, problems 23, 24, 26, 43, and 46.

Post-Assessment VII. Do all three tasks.

Task I. Mrs. Jones wants to cover the two walls of her room with wallpaper. One wall is 8' 10" by 15'. The other wall is 8' 10" by 24'. There are four windows that are each 3' by 60". The wallpaper Mrs. Jones chose does not have a pattern that must match on each panel. Each roll of wallpaper is 28" wide and 15 yards long.

How many rolls will she need?

Answer_____

Task II. A rectangular sheet of parchment 37" long and 18" wide is to be rolled to form a cylindrical lampshade. There is a one inch overlap for joining and seaming. The lampshade is 18" high. From this information is it possible to find the circumference (perimeter) of the base of the lampshade and the surface area of the lampshade?

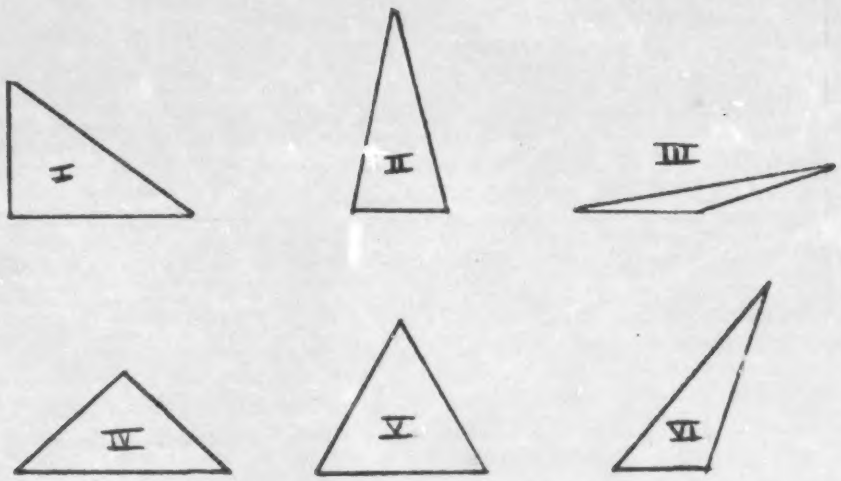
Answer_____

If yes, find the circumference (perimeter) and the surface area.

If no, explain what other information is needed.

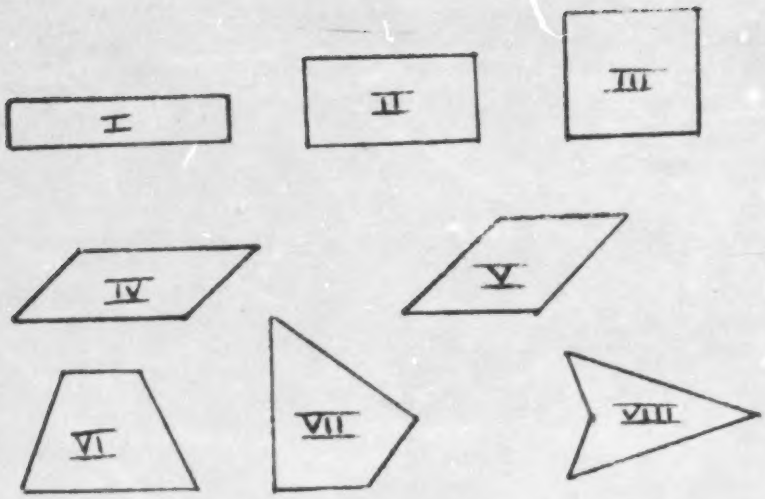
Task III. Find the perimeter and area of each of the figures below by making appropriate measurements or by tracing the figures on graph paper (use ¼ inch graph paper) and counting appropriate units. The linear unit to be used is ⊢⊣ (a quarter inch) and the area unit to be used is ▪ (a quarter inch square). Record your results on Table VII on page 176.

Triangles

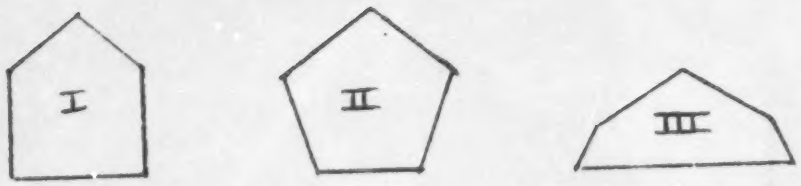


TURN TO NEXT PAGE.

Quadrilaterals

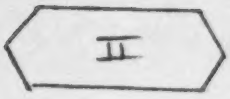


Pentagons

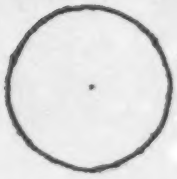


TURN TO NEXT PAGE.

Hexagons



Circle



TURN TO NEXT PAGE.

Table VII

	Perimeter	Approximate Area in Square Units
Triangle: I II III IV V VI		

Which triangle has the largest area?_____
What are the characteristics of this triangle?_____

Quadrilateral: I II III IV V VI VII VIII		

Which quadrilateral has the largest area?_____
What are the characteristics of this quadrilateral?_____

Pentagon: I II III		

Which pentagon has the largest area?_____
What are the characteristics of this pentagon?_____

Hexagon: I II		
Circle:		

TURN TO NEXT PAGE.

a. Compare the perimeters of all the figures in this task. What do you notice?

_____

b. Compare the areas of all the figures in this task. What do you notice?

_____

c. Which plane figure has the largest area in each category?

_____

d. When you consider all the categories, which plane figures with perimeter of 12 units has the largest area?

_____

e. On the basis of the above data, formulate three hypothesis that the data suggest.

Hypothesis 1_____

_____

_____

Hypothesis 2_____

_____

_____

Hypothesis 3_____

_____

_____

TURN TO NEXT PAGE.

f. Describe what further experimentation or exploration you would do to test your hypotheses.

TURN TO THE NEXT PAGE FOR FURTHER DIRECTIONS AND DISCUSSION OF RESPONSES TO ALL THE TASKS.

Directions

1. Check your responses to the post-assessment tasks on pages 17-18 of the Response Manual.

2. Your responses to

   a. Task I agree? Yes____

      No_____

   b. Task II agree? Yes_____

      No_____

   c. Task III agree? Yes_____

      No_____

3. If you have checked No to any part of the above questions, read one of the suggested readings on page 171 and redo post-assessment task. Then go on to Direction 4.

4. If you have checked Yes to all parts of question 2, go back to page 10.

5. On your Decision Chart, place a check in column 3 to indicate that you have completed the original activity.

6. Now turn to Section II and complete Activity D on pages 201-205.

GLOSSARY OF TERMS

A simple closed curve is a set of points that can be traced without lifting pencil from the paper so that the curve starts and ends at the same point without ever crossing or retracing itself. Thus a triangle is a simple closed curve.

A plane region is the union of a simple closed curve and its interior.

The measure of a geometric object (line segment, perimeter, plane region) in terms of a unit is the number (not necessarily a whole number) of times the unit will fit into the object.

The measure of a region is called its area.

The perimeter of a plane figure is the length of the simple closed curve which forms the figure.

Two geometric figures are congruent if they can be made to coincide.

A polygon is a simple closed curve which is the union of three or more straight line segments. The straight line segments are called sides of the polygon.

A triangle is a polygon of three sides.

A quadrilateral is a polygon of four sides.

A pentagon is a polygon of five sides.

A hexagon is a polygon of six sides.

An octagon is a polygon of eight sides.

A parallelogram is a quadrilateral with two pairs of opposite sides parallel.

A rectangle is a parallelogram with one right angle.

A rhombus is a parallelogram with two adjacent sides congruent.

A square is a rectangle with two adjacent sides congruent.

A trapezoid is a quadrilateral with two and only two sides parallel.

A regular polygon is a polygon which is both equilateral and equiangular.

Some properties of triangles:

An isosceles triangle has two congruent sides.

An equilateral triangle has three congruent sides.

A scalene triangle has no sides congruent.

A median of a triangle is a line segment joining a vertex of a triangle to the midpoint of the opposite side.

A circle is the set of all points in a plane at a given distance from a fixed point.

The perimeter (circumference) of a circle is approximately equal to three times the length of its diameter. $C = \pi d$ (See Sadlier, Contemporary Mathematics 7, p. 374, ex. 6-10)

A circular region is the union of a circle and its interior.

A sphere is the set of all points in space at a given distance from a fixed point.

A great circle of a sphere is the intersection of the sphere and a plane passing through the center of the sphere. (Example: on a globe, the Equator and all longitudinal circles are great circles.)

A circle formed by the intersection of a sphere and a plane not passing through the center of the sphere is called a small circle of the sphere. (Example: On a globe, latitudinal circles other than the Equator are small circles.)

Section II

The activities in this section are designed to assist you in preparing sequences of learning tasks (relating to the concept of area) for ten to twelve year old students.

What is a learning task? A learning task is an experience which facilitates insight into one or more aspects of an objective. For example, learning task 1.1 in Section I is designed to facilitate insight into one aspect of an objective, namely, the recognition of two-dimensional regions. There are three integral parts to the learning experience:

1. The student must be given something to work with--the picture of a room.

2. The student must be requested to do something--to identify the plane regions in the picture.

3. The student must be required to record his response in order that his experiences might be assessed--a written list of his responses (see page 19).

What is a sequence of learning tasks? A sequence of learning tasks is an ordered collection of learning tasks that are appropriate for the attainment of the objective. For example, Element I, in Section 1, is a sequence of learning tasks that are designed to attain the objective - given a set of representative materials, you classify the materials according to their appropriateness for measuring 1, 2, or 3 dimensional space and justify the material selection for measurement of two-dimensional space.

Element I, requires two learning tasks--the identification of a plane region (Task 1.1) and the differentiation between one- and two-dimensional measurement (Task 1.2). Clearly, Task 1.1 precedes Task 1.2 because you must be able to identify a plane region before you can differentiate between the measurement going around a plane region and measurement covering the plane region.

In the planning of your tasks, you should keep in mind the principles of developmental psychology as expressed by Jean Piaget and others - specifically, that one moves from the concrete to the abstract. All of the tasks in Element II utilize concrete materials to develop an intuitive understanding of tessellation, i.e., the covering of a plane region. For Task 2.1, familiar geometric shapes are used to introduce the concept of tessellation while Task 2.2 allows the student to develop his creativity in discovering

"new" geometric shapes which will tessellate.

NOW DO THE FOLLOWING EXERCISE

1. Go back and look at Task 3.1 and Task 3.2. What would be your rationale for Task 3.1 preceding Task 3.2?

_____

_____

_____

_____

TURN TO PAGE 187 AND CHECK YOUR RESPONSES.

2. Go back and look at Task 3.3 and Task 3.4. What would be your rationale for Task 3.3 preceding Task 3.4?

_____

_____

_____

_____

TURN TO PAGE 187 AND CHECK YOUR RESPONSES.

3. As a teacher you must frequently use several different concrete materials in developing a concept. In Tasks 3.3 and 3.4, a grid was used in developing the idea of standard unit of measure. List two other materials you think appropriate in developing this concept.

_____

_____

TURN TO PAGE 187 AND CHECK YOUR RESPONSES.

Discussion of Responses

1. Task 3.1 precedes Task 3.2 because Task 3.1 deals with the identification of unit of measure and measurement of a plane region involving two dimensions while Task 3.2 deals with measurement of a plane region involving three dimensions.

NOW TURN BACK TO PAGE 186 AND DO QUESTION NUMBER 2.

2. Task 3.3 precedes Task 3.4 because Task 3.3 introduces the use of grids for measuring plane region and Task 3.4 extends the use of a grid to standard unit of measure.

NOW TURN BACK TO PAGE 186 AND DO QUESTION 3.

3. Various materials may be used to help develop the concept. For example--tiles.

You will find it helpful to examine fifth- or sixth-grade mathematics textbooks and to analyze how the authors have set up sequences of learning tasks to attain certain objectives in relation to student understanding of the concept of area.

NOW TURN TO THE NEXT PAGE FOR THE DIRECTIONS FOR SECTION II.

Directions for Section II

Do all the activities in this section.

a. Activity A should be done after you have reviewed Elements 1, 2, and 3 of Section I.

b. Activity B should be done after you have reviewed Elements 4 and 5 of Section I.

c. Activity C should be done after you have reviewed Element 6 of Section I.

d. Activity D should be done after you have reviewed Element 7 of Section I.

ACTIVITY A

Objective: Given a specific objective related to the study of area, you will prepare a satisfactory sequence of learning tasks that can be developed into a set of lessons for ten- to twelve-year-old students.

Before beginning this activity, you may wish to reread the discussion of learning tasks and sequencing at the beginning of this section, page 185 . Review the objectives of Element I, page 15, Element II, page 28, and Element III, page 41, and keep them in mind while doing this activity. When you have finished writing in your plans and suggestions, turn to page 194 and check your responses. You might also want to refer to some of the books listed in Activity 2 of each element in Section I. Now begin.

PREPARATION OF A SEQUENCE OF LEARNING TASKS FOR THE GIVEN OBJECTIVE.

OBJECTIVE:

_____

_____

_____

_____

SEQUENCE OF LEARNING TASKS:

(If you have more than three tasks submit them on additional paper)

Task 1

CONTINUE ON NEXT PAGE.

Task 2

Task 3

TURN TO NEXT PAGE.

Discussion of Responses

Using the questions listed below, analyze your sequence of learning tasks.

CHECKLIST

Check your response in the appropriate space:	Yes	No
1. Did you state your objective?	___	___
2. Do you have a specific learning task for each of the enabling objectives?	___	___
3. Does each learning task state what the student is given to work with?	___	___
4. Does each task state what the student will be required to do?	___	___
5. Have you stated what the student will record in order that you might assess his learning experience?	___	___

Directions

1. If you have answered "yes" to all the questions, submit your sequence of learning tasks to your instructor for assessment.

2. Follow the directions of your instructor. If your instructor deems your sequence of learning tasks satisfactory, proceed to the next circled item on your Decision Chart on page 10.

3. If you could not answer "yes" to all the questions review the explanation given for the development of a sequence of learning tasks and redo.

4. Submit your sequence of learning tasks to your instructor for assessment.

5. Follow the directions of your instructor.

ACTIVITY B

Objective: Given a specific objective related to the study of area, you will prepare a satisfactory sequence of learning tasks that can be developed into a set of lessons for ten- to twelve-year-old students.

Before beginning this activity, you may wish to reread the discussion of learning tasks and sequencing at the beginning of this section, pages 185-186. Review the objectives of Element IV, page 63, and Element V, page 92, and keep them in mind while doing this activity. When you have finished writing in your plans and suggestions, turn to page 194 and check your responses. You might also want to refer to some of the books listed in Activity 2 of each element in Section I. Now begin.

PREPARATION OF A SEQUENCE OF LEARNING TASKS FOR THE GIVEN OBJECTIVE.

OBJECTIVE:

_____

_____

_____

_____

SEQUENCE OF LEARNING TASKS:

(If you have more than two tasks, submit them on additional pages)

Task 1

CONTINUE ON NEXT PAGE.

Task 2

TURN TO NEXT PAGE.

Discussion of Responses

Using the questions listed below, analyze your sequence of learning tasks.

CHECKLIST

Check your response in the appropriate space:	Yes	No
1. Did you state your objective?	___	___
2. Do you have a specific learning task for each of the enabling objectives?	___	___
3. Does each learning task state what the student is given to work with?	___	___
4. Does each task state what the student will be required to do?	___	___
5. Have you stated what the student will record in order that you might assess his learning experience?	___	___

Directions

1. If you have answered "yes" to all the questions, submit your sequence of learning tasks to your instructor for assessment.

2. Follow the directions of your instructor. If your instructor deems your sequence of learning tasks satisfactory, proceed to the next circled item on your Decision Chart on page 10 .

3. If you could not answer "yes" to all the questions review the explanation given for the development of a sequence of learning tasks and redo.

4. Submit your sequence of learning tasks to your instructor for assessment.

5. Follow the directions of your instructor.

ACTIVITY C

Objective: Given a specific objective related to the study of area, you will prepare a satisfactory sequence of learning tasks that can be developed into a set of lessons for ten- to twelve-year-old students.

Before beginning this activity, you may wish to reread the discussion of learning tasks and sequencing at the beginning of this section, pages 185-186. Review the Objectives of Element VI page 119, and keep them in mind while doing this activity. When you have finished writing in your plans and suggestions, turn to page 200 and check your responses. You might also want to refer to some of the books listed in Activity 2 of each element in Section I. Now begin.

This activity is slightly different from Activities A and B.

Task C-1

In Element VI, the tasks required you to make observations, to describe and analyze what you noticed, to use various materials and construct figures, to recognize patterns and to formulate hypotheses. Now you are to consult two fifth- or sixth-grade elementary mathematics textbooks and examine lessons on the concept of area. Select a lesson from each textbook and for each lesson determine what its objectives are. Then outline how you would use each lesson with children, noting particularly:

a. what activities you would involve children in;

b. what questions you would ask in order to have the children make observations, describe and analyze their observations, construct figures, recognize patterns and formulate hypotheses; and

c. what real world applications of the concepts you would include which would be of interest to the children.

Lesson from First Textbook:

Name of Text:_____pages_____

Objectives of Lesson:_____

_____

_____

CONTINUE ON NEXT PAGE.

Outline below how you would use this lesson with children (Note a, b, c above).

Lesson from Second Textbook:

Name of Text:_____ pages_____

Objectives of Lesson:_____

_____

_____

CONTINUE ON NEXT PAGE.

Outline below how you would use this lesson with children
(Note a, b, c above)

BEGIN TASK C-2 ON NEXT PAGE.

Task C-2: PREPARATION OF A SEQUENCE OF LEARNING TASKS FOR THE GIVEN OBJECTIVE.

OBJECTIVE:

_____

_____

_____

_____

SEQUENCE OF LEARNING TASKS:

(If you have more than two tasks, submit them on additional pages)

Task 1

CONTINUE ON NEXT PAGE.

Task 2

CONTINUE ON NEXT PAGE.

Discussion of Responses

Using the questions listed below, analyze your sequence of learning tasks.

CHECKLIST

Check your response in the appropriate space:

	Yes	No
1. Did you state your objective?	___	___
2. Do you have a specific learning task for each of the enabling objectives?	___	___
3. Does each learning task state what the student is given to work with?	___	___
4. Does each task state what the student will be required to do?	___	___
5. Have you stated what the student will record in order that you might assess his learning experience?	___	___

Directions

1. If you have answered "yes" to all the questions, submit your sequence of learning tasks to your instructor for assessment. Also submit your responses to Task C-1 to your instructor.

2. Follow the directions of your instructor. If your instructor deems your sequence of learning tasks satisfactory, proceed to the next circled item on your Decision Chart on page 10.

3. If you could not answer "yes" to all the questions, review the explanation given in this activity for the development of a sequence of learning tasks and redo

4. Submit your sequence of learning tasks to your instructor for assessment.

5. Follow the directions of your instructor.

ACTIVITY D

Objective: Given a specific objective related to the study of area, you will prepare a satisfactory sequence of learning tasks that can be developed into a set of lessons for ten- to twelve-year-old students.

Before beginning this activity, you may wish to reread the discussion of learning tasks and sequencing at the beginning of this section, pages 185-186. Review the objectives of Element VII, page 161, and keep them in mind while doing this activity. When you have finished writing in your plans and suggestions, turn to page and check your responses. You might also want to refer to some of the books listed in Activity 2 of each element in Section I. Now begin.

Task D-1

Consider the following problem-solving situation:

In pioneer days when land was plentiful and almost everything else was very scarce, a man in the Midwest had hundreds of acres of flat grassland, but only 120 yards of barbed wire fencing. He intended to use the whole length of this wire to fence in a piece of his land. He thought of various shapes and wondered how many square yards would be in the fenced-in area. Noting the last sentence and thinking of a sixth-grade class, indicate the following:

a. How the children might assist in the formulation of the problem.

b. What additions you (as teacher) might make to the children's suggestions.

CONTINUE ON NEXT PAGE.

c. What initial hypotheses might be formulated?

d. How actual testing of hypotheses might proceed and what results might be obtained.

CONTINUE ON NEXT PAGE.

e. What insights will emerge from the patterns and results noted:

BEGIN TASK D-2 ON THE NEXT PAGE.

Task D-2 PREPARATION OF A SEQUENCE OF LEARNING TASKS FOR THE GIVEN OBJECTIVE

OBJECTIVE:

_____

_____

_____

_____

SEQUENCE OF LEARNING TASKS

(If you have more than two tasks, submit them on additional pages)

Task 1

CONTINUE ON NEXT PAGE.

Task 2

TURN TO NEXT PAGE.

CHECKLIST

Check your response in the appropriate space:	Yes	No
1. Did you state your objective?	___	___
2. Do you have a specific learning task for each of the enabling objectives?	___	___
3. Does each learning task state what the student is given to work with?	___	___
4. Does each task state what the student will be required to do?	___	___
5. Have you stated what the student will record in order that you might assess his learning experience?	___	___

Directions

1. If you have answered "yes" to all the questions, submit your sequence of learning tasks to your instructor for assessment. Also submit your responses to Task D-1 to your instructor.

2. Follow the directions of your instructor. If your instructor deems your tasks satisfactory, place a check in the appropriate column on your Decision Chart on page 10 and proceed to Section III.

3. If you could not answer "yes" to all the questions review the explanation given for the development of a sequence of learning tasks and redo.

4. Submit your sequence of learning tasks to your instructor for assessment.

5. Follow the directions of your instructor.

Section III

This section is designed to enable you to present and assess one of the sequences of learning tasks which you developed in Section II.

When you have completed this section you should be able to meet the following objective:

> Given a group of ten-to twelve-year-old children you will present a satisfactory sequence of learning tasks on the concept of area and assess the appropriateness of these tasks in light of student learning.

Directions:

1. See your instructor. Your instructor will select one of the sequence of learning tasks that you have developed in Section II for you to present to a group of ten-to twelve-year-old students. Your instructor will discuss with you what evidence of learning should be collected as part of your assessment of the lesson.

2. Your instructor will assign you to a group of ten-to twelve-year-old students to whom you are to present and assess your sequence of learning tasks as a mini lesson.

3. Your instructor will provide you with a criteria list with which you are to assess your presentation of the mini lesson.

4. You will present your lesson to the group of students.

5. You will assess your lesson which you have given to these students.

6. Present your assessment of your lesson and the evidence of learning which you have collected as part of the assessment to your instructor.

7. Consult with your instructor as to whether or not you may exit the module.